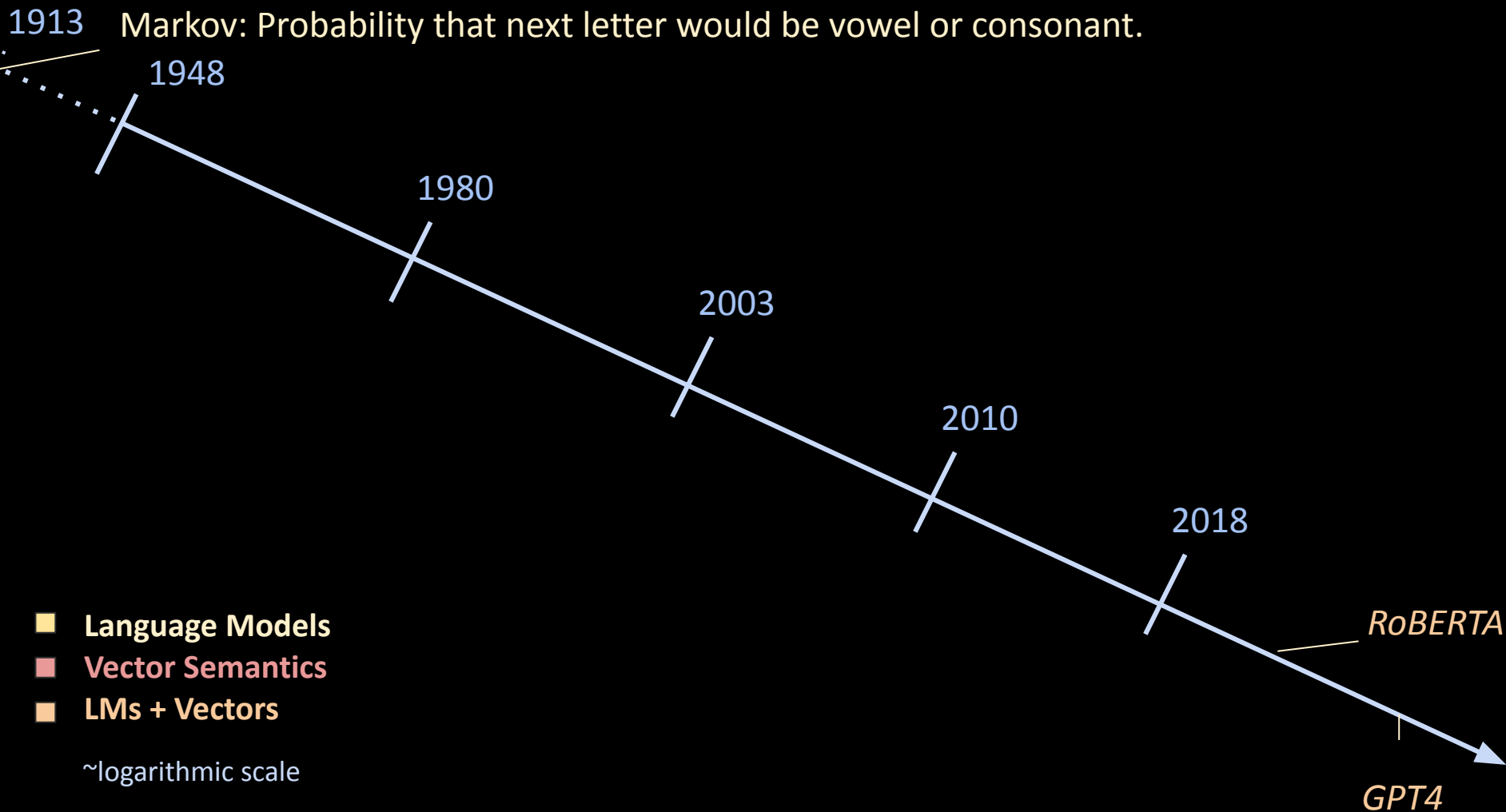


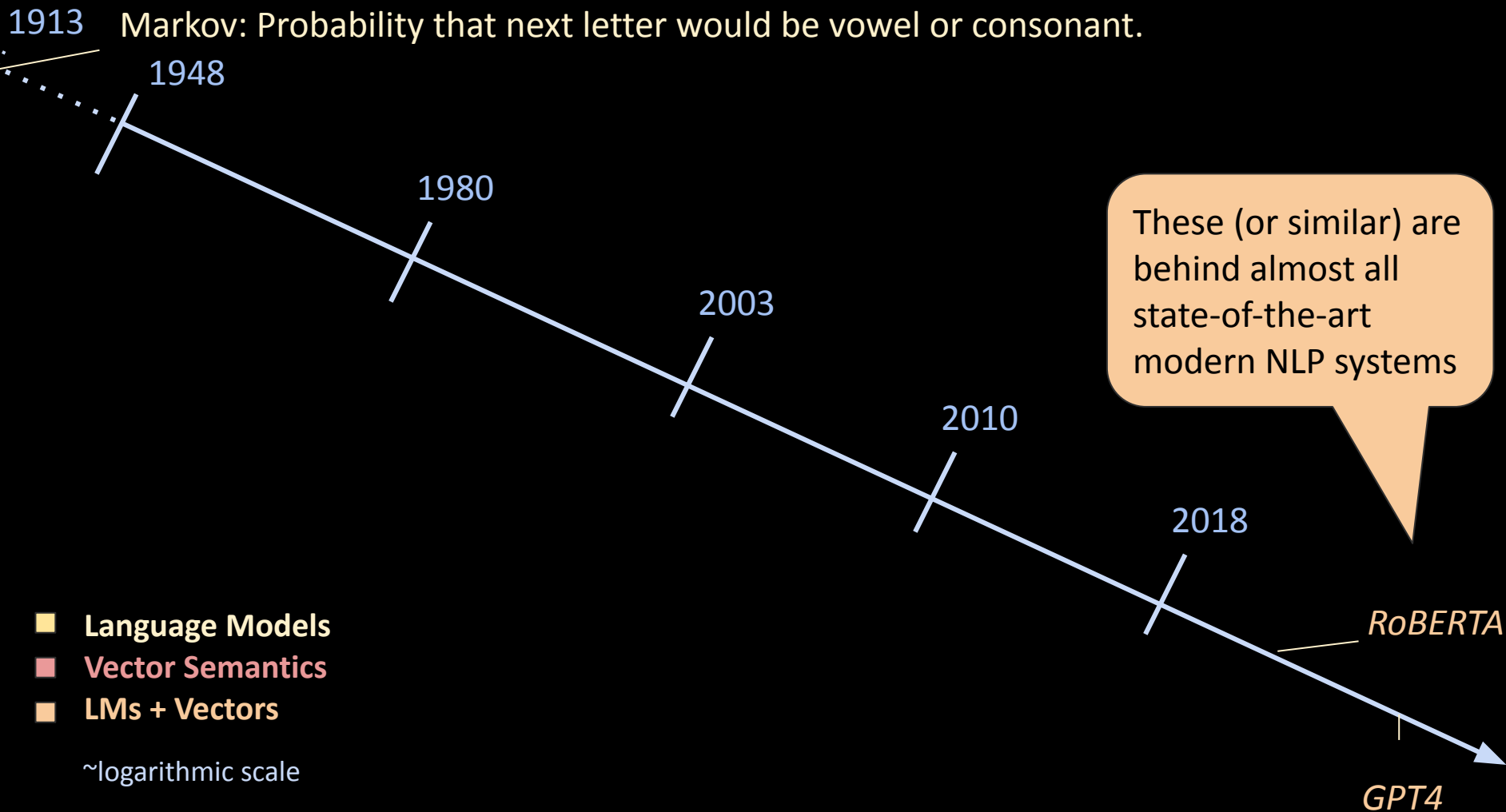
# Transformer Language Models



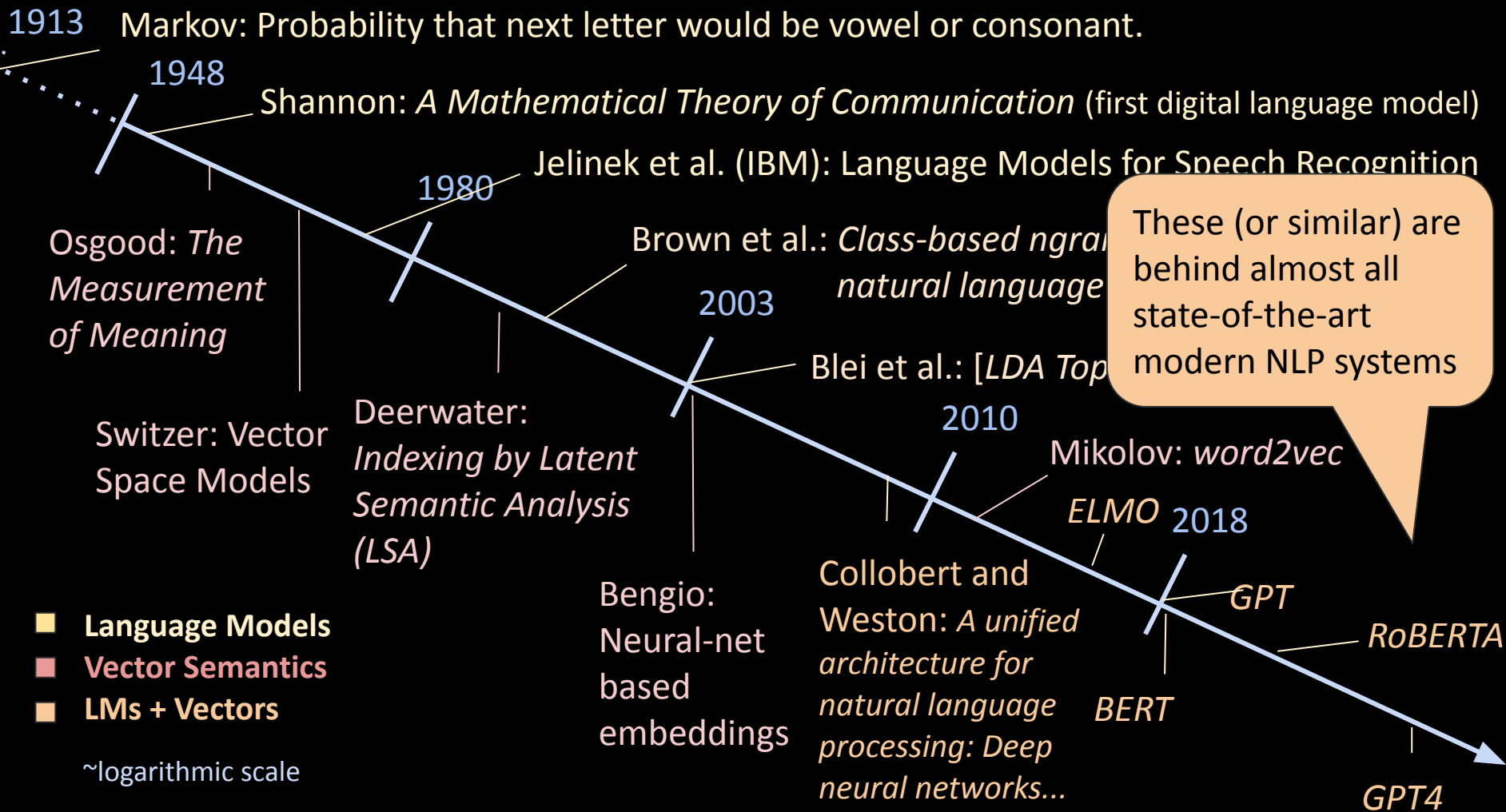
# Timeline: *Language Modeling* and *Vector Semantics*



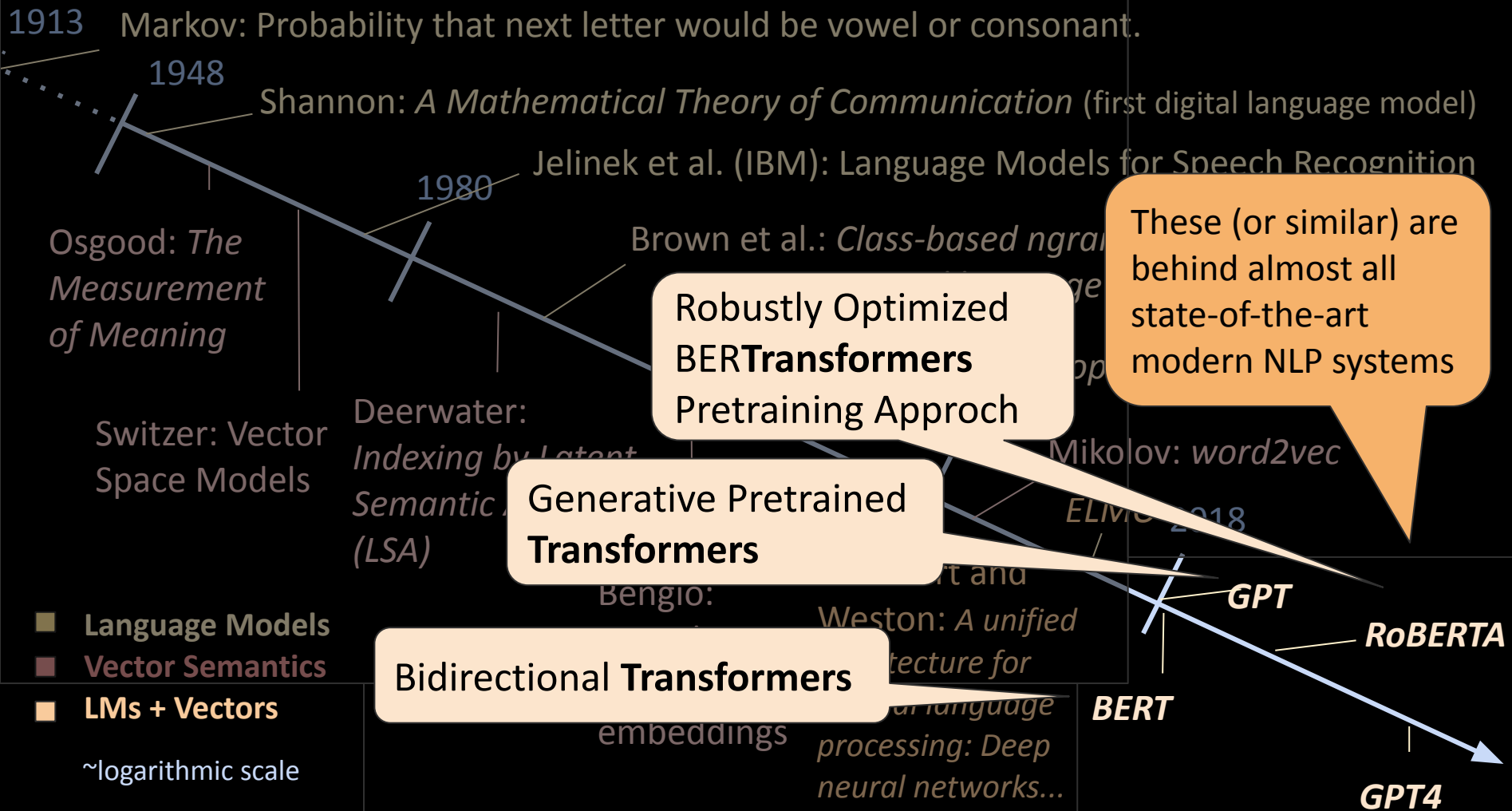
# Timeline: *Language Modeling* and *Vector Semantics*



# Timeline: *Language Modeling* and *Vector Semantics*



# Timeline: *Language Modeling* and *Vector Semantics*



# Transformers

## Attention Is All You Need

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illia.polosukhin@gmail.com

### Abstract

The dominant sequence transduction models are based on complex recurrent or convolutional neural networks that include an encoder and a decoder. The best performing models also connect the encoder and decoder through an attention mechanism. We propose a new simple network architecture, the Transformer, based solely on attention mechanisms, dispensing with recurrence and convolutions entirely. Experiments on two machine translation tasks show these models to be superior in quality while being more parallelizable and requiring significantly less time to train. Our model achieves 28.4 BLEU on the WMT 2014 English-to-German translation task, improving over the existing best results, including the state-of-the-art single-model state-of-the-art BLEU score of 41.0 after scaling up over 2 BLEU. On the WMT 2014 English-to-French translation task, our model achieves 40.6 BLEU, improving over the existing best results, including the state-of-the-art single-model state-of-the-art BLEU score of 41.0 after scaling up over 2 BLEU. Our model achieves a small fraction of the training costs of the

Vaswani, A., Shazeer, N.,  
Parmar, N., Uszkoreit, J.,  
Jones, L., Gomez, A. N., ... &  
Polosukhin, I. (2017). Attention  
is all you need. *Advances in neural  
information processing systems*, 30.

# Transformers



## Self-Attention



## Deep Learning



## Neural Networks

# Transformers



## Self-Attention



## Deep Learning



## Neural Networks



- multi-headed attention
  - positional embeddings
  - residual links
- (to be introduced later)



# Part 1: Deep Learning and Masked Language Models

Adithya V Ganesan

CSE538 - Spring 2024  
[bit.ly/cse538-sp24-lecture7](https://bit.ly/cse538-sp24-lecture7)

# Artificial Neural Networks

*What is it?*

# Artificial Neural Networks

*What is it?*

- Biologically inspired computing model
- Learn patterns from the data
- Can even approximate nonlinear functions in the nature!

# Artificial Neural Networks

*What is it?*

- Biologically inspired **computing model**
- Learn patterns from the data
- Can even approximate nonlinear functions in the nature!

# Artificial Neural Networks

*What is it?*



*How did we do this?*

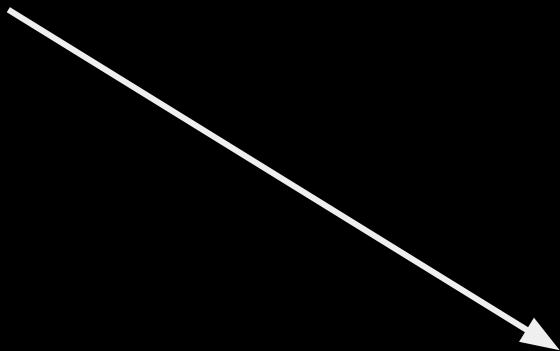
- Biologically inspired **computing model**
- Learn patterns from the data
- Can even approximate nonlinear functions in the nature!

But, how do we model complex systems using these linear systems?

# Deep Learning

But, how do we model complex systems using these linear systems?

# Deep Learning



Non-linear functions + Artificial Neural Networks



# Activation Functions

$$z = h_{(t)}W$$

# Common Activation Functions

$$z = h_{(t)}W$$

Logistic:  $\sigma(z) = 1 / (1 + e^{-z})$

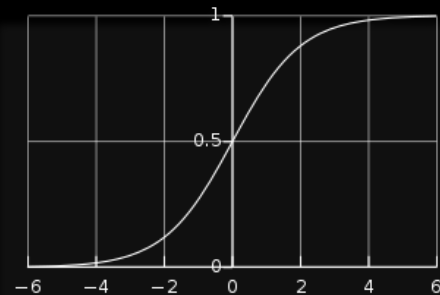
Hyperbolic tangent:  $\tanh(z) = 2\sigma(2z) - 1 = (e^{2z} - 1) / (e^{2z} + 1)$

Rectified linear unit (ReLU):  $ReLU(z) = \max(0, z)$

# Common Activation Functions

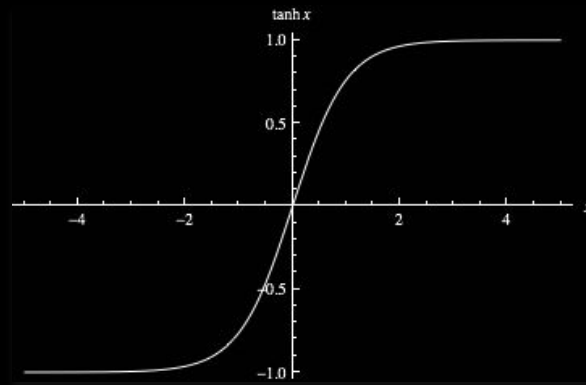
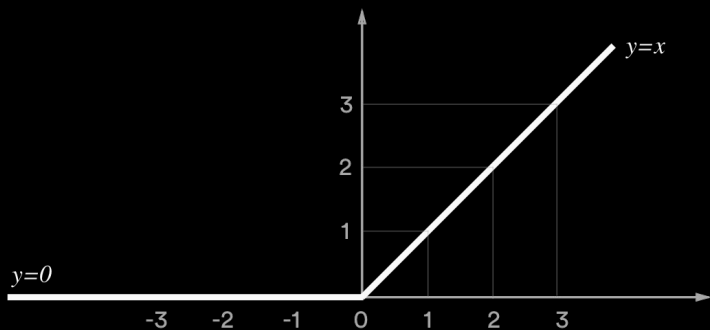
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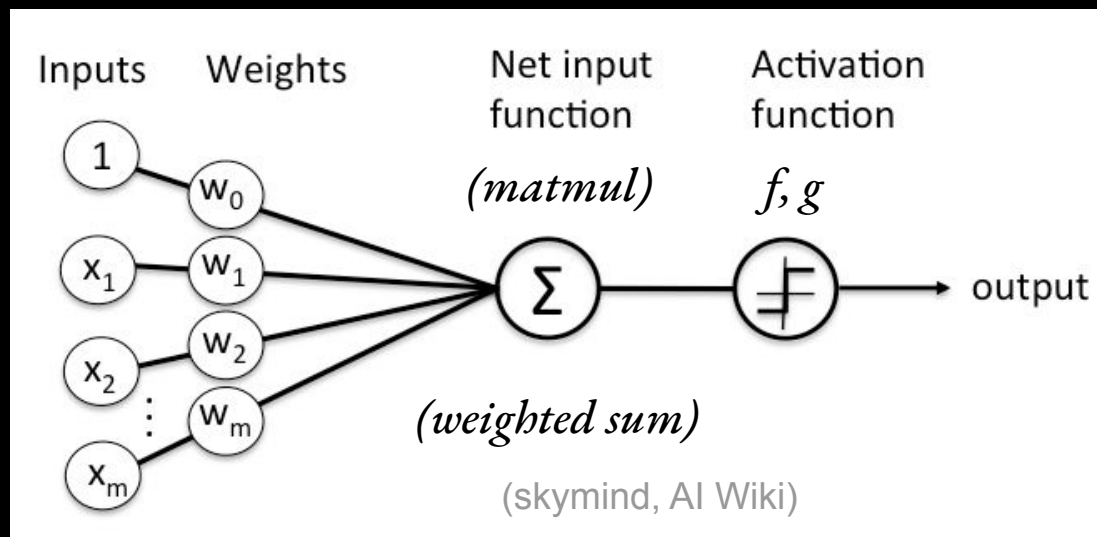


Hyperbolic tangent:  $\tanh(z) = 2\sigma(2z) - 1 = (e^{2z} - 1) / (e^{2z} + 1)$

Rectified linear unit (ReLU):  $ReLU(z) = \max(0, z)$



# Neural Networks: Graphs of Operations (excluding the optimization nodes)

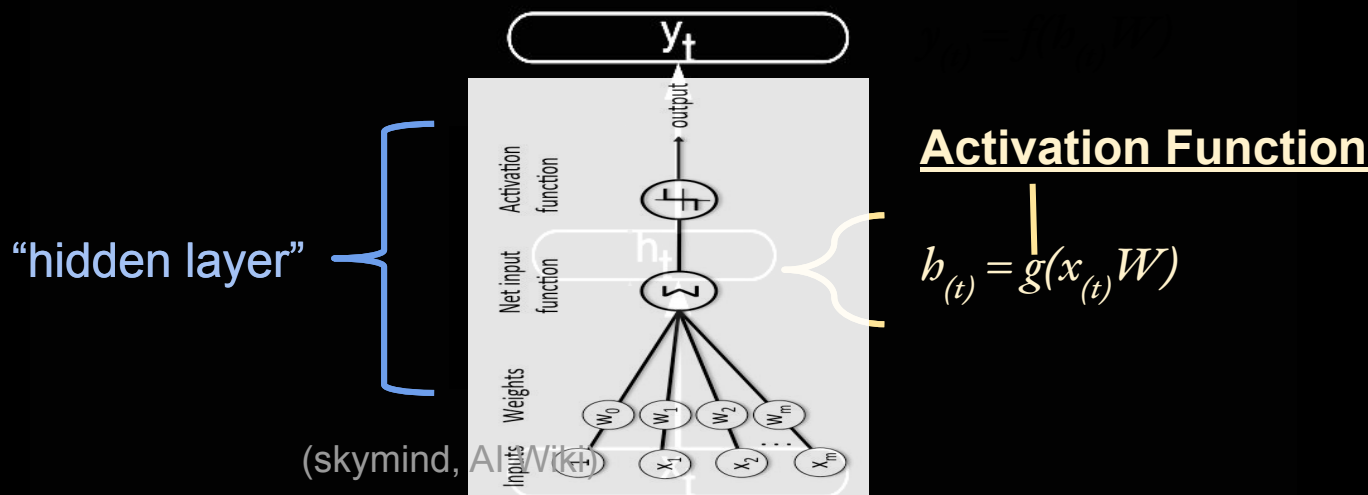


## Activation Function

$$h_{(t)} = g(x_{(t)}W)$$

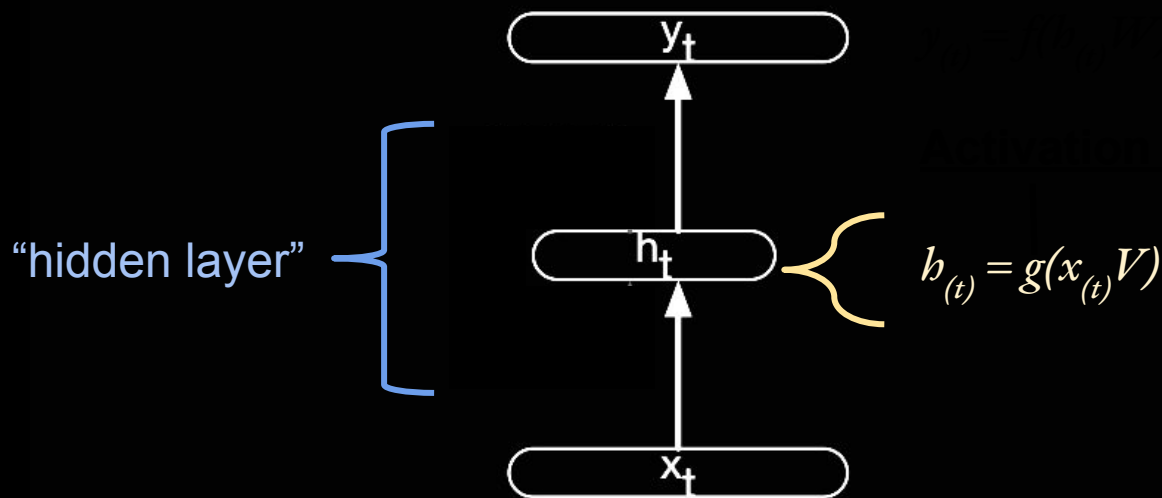
**Figure 9.2** Simple recurrent neural network after Elman (Elman, 1990). The hidden layer includes a recurrent connection as part of its input. That is, the activation value of the hidden layer depends on the current input as well as the activation value of the hidden layer from the previous timestep. (Jurafsky, 2019)

# Neural Networks: Graphs of Operations (excluding the optimization nodes)



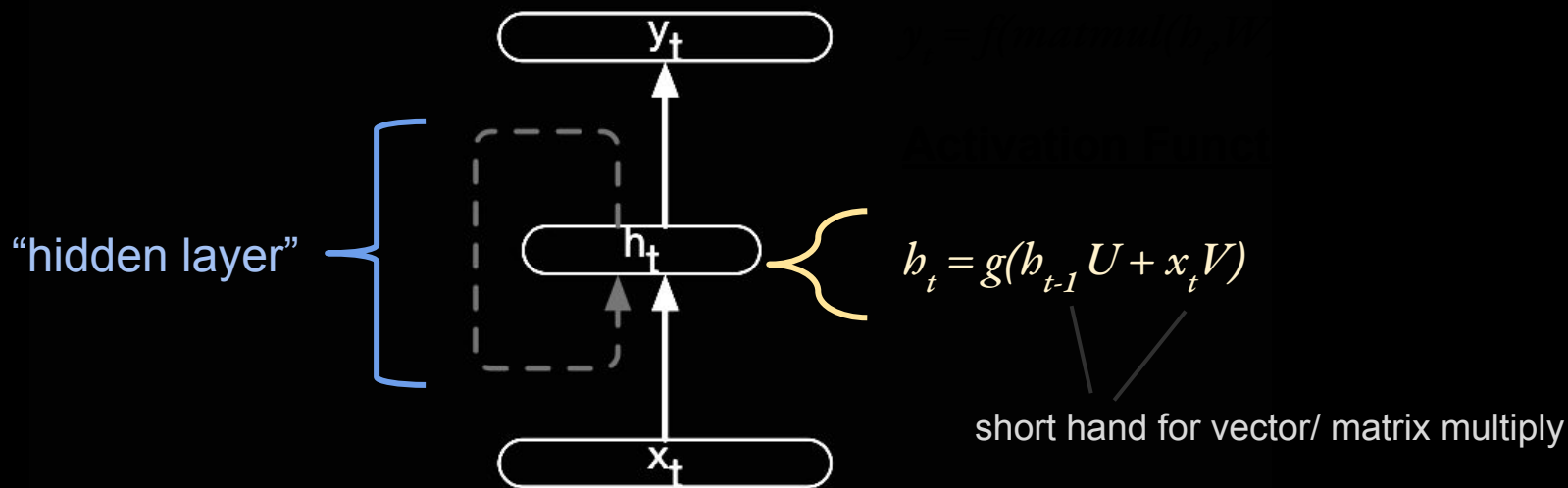
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# Neural Networks: Graphs of Operations (excluding the optimization nodes)



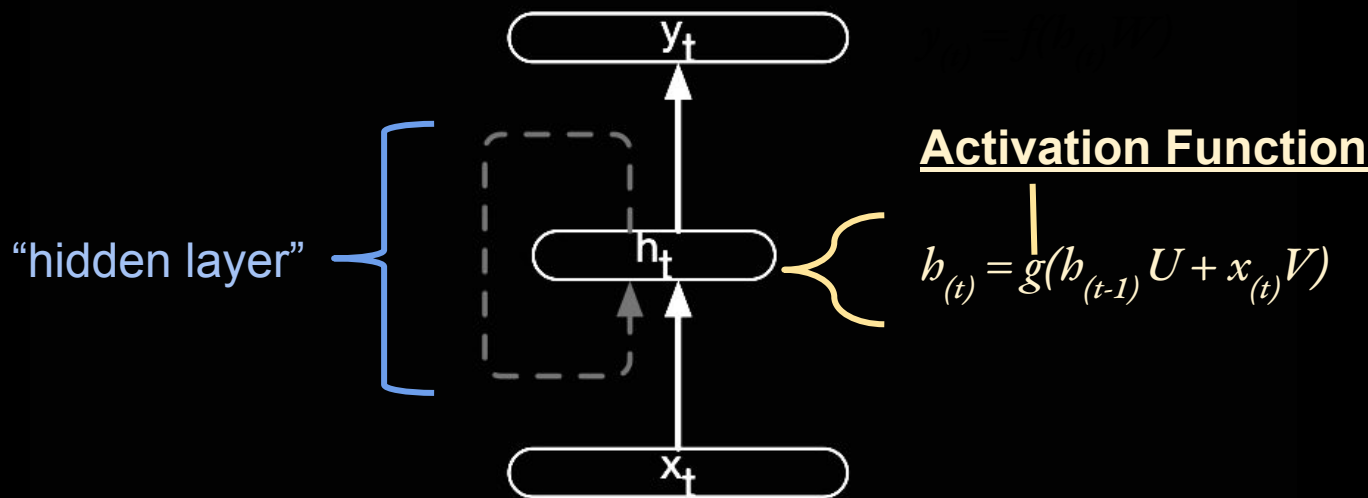
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# Neural Networks: Graphs of Operations (excluding the optimization nodes)



**Figure 9.2** Simple recurrent neural network after Elman (Elman, 1990). The hidden layer includes a recurrent connection as part of its input. That is, the activation value of the hidden layer depends on the current input as well as the activation value of the hidden layer from the previous timestep. (Jurafsky, 2019)

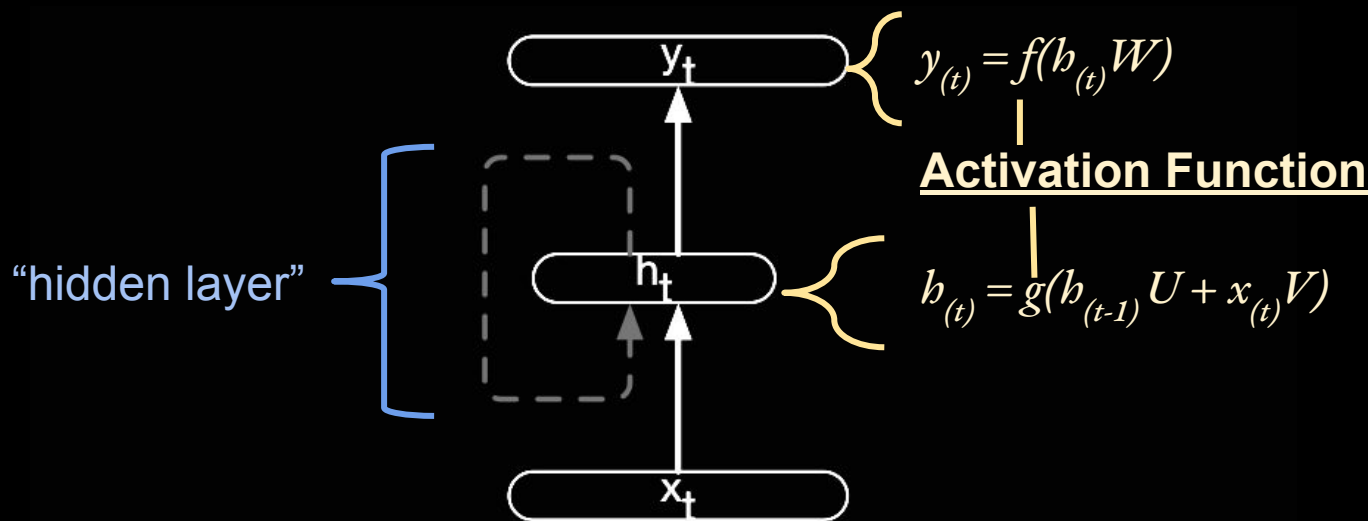
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# Back Propagation

# Timeline: *Language Modeling* and *Vector Semantics*

1913 Markov: Probability that next letter would be vowel or consonant.

1948

Shannon: *A Mathematical Theory of Communication* (first digital language model)

Osgood: *The Measurement of Meaning*



- Language Models
- Vector Semantics
- LMs + Vectors

~logarithmic scale

(Li et al., 2015; Jurafsky et al., 2019)

XLNet  
RoBERTA

GPT4

# Word Vectors

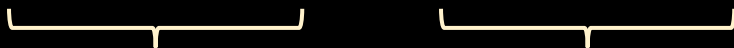
To embed: convert a token (or sequence) to a vector that represents **meaning**.

Wittgenstein, 1945: “*The meaning of a word is its use in the language*”

Distributional hypothesis -- A word's meaning is defined by all the different contexts it appears in (i.e. how it is “distributed” in natural language).

Firth, 1957: “*You shall know a word by the company it keeps*”

*The nail hit the beam behind the wall.*



# Word Vectors

## Person A

*How are you?*

I feel *fine* —even *great*!

*What is going on?*

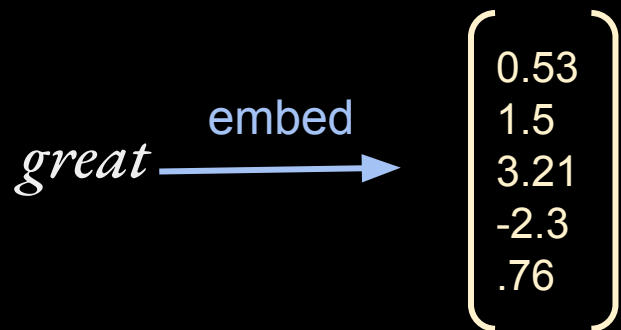
Earlier, I *played* the *game* Yahtzee with my *partner*. I could not get that *die* to roll a 1! Now I'm *lying* on my bed for a *rest*.

## Person B

My life is a *great* mess! I'm having a very hard time being happy.

My business *partner* was *lying* to me. He was trying to *game* the system and *played* me. I think I am going to *die* —he left and now I have to pay the *rest* of his *fine*.

# Objective



# Objective

*great* → embed

$$\begin{bmatrix} 0.53 \\ 1.5 \\ 3.21 \\ -2.3 \\ .76 \end{bmatrix}$$

?

**great.a.1** (relatively large in size or number or extent; larger than others of its kind)

**great.a.2**, outstanding (of major significance or importance)

**great.a.3** (remarkable or out of the ordinary in degree or magnitude or effect)

bang-up, bully, corking, cracking, dandy, **great.a.4**, groovy, keen, neat, nifty, not bad, peachy, slap-up, swell, smashing, old (very good)

capital, **great.a.5**, majuscule (uppercase)

big, enceinte, expectant, gravid, **great.a.6**, large, heavy, with child (in an advanced stage of pregnancy)

# Objective

*great* → embed

$$\begin{bmatrix} 0.53 \\ 1.5 \\ 3.21 \\ -2.3 \\ .76 \end{bmatrix}$$

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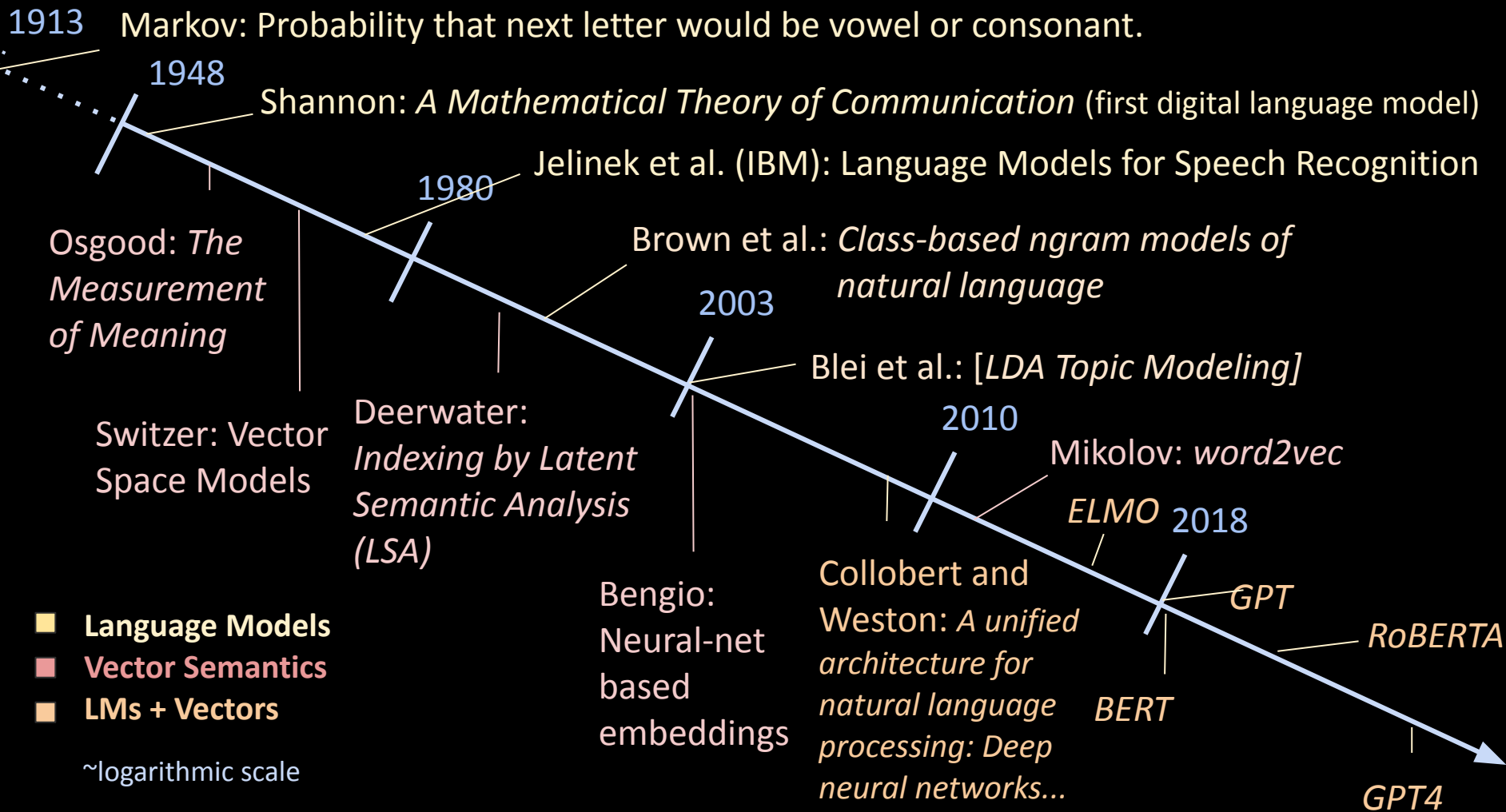
capital, **great.a.5**, majuscule (uppercase)

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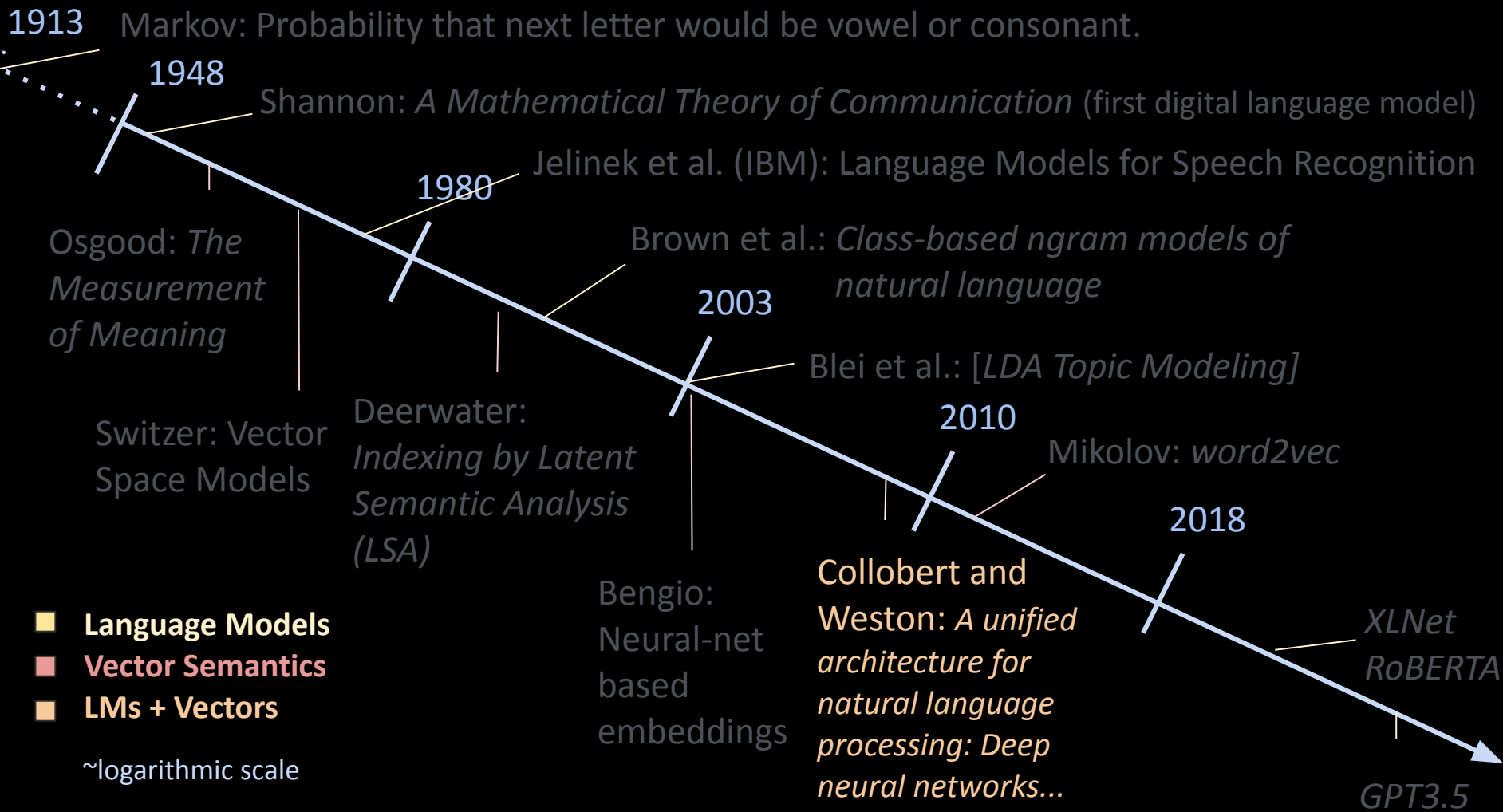
**great.n.1** (a person who has achieved distinction and honor in some field)



# Timeline: *Language Modeling* and *Vector Semantics*



# Timeline: *Language Modeling* and *Vector Semantics*



# Modeling and Vector Semantics

r would be vowel or consonant.

*l Theory of Communication* (first digital language model)

et al. (IBM): Language Models for Speech Recognition

rown et al.: *Class-based ngram models of natural language*

2003

Blei et al.: [LDA Topic Modeling]

2010

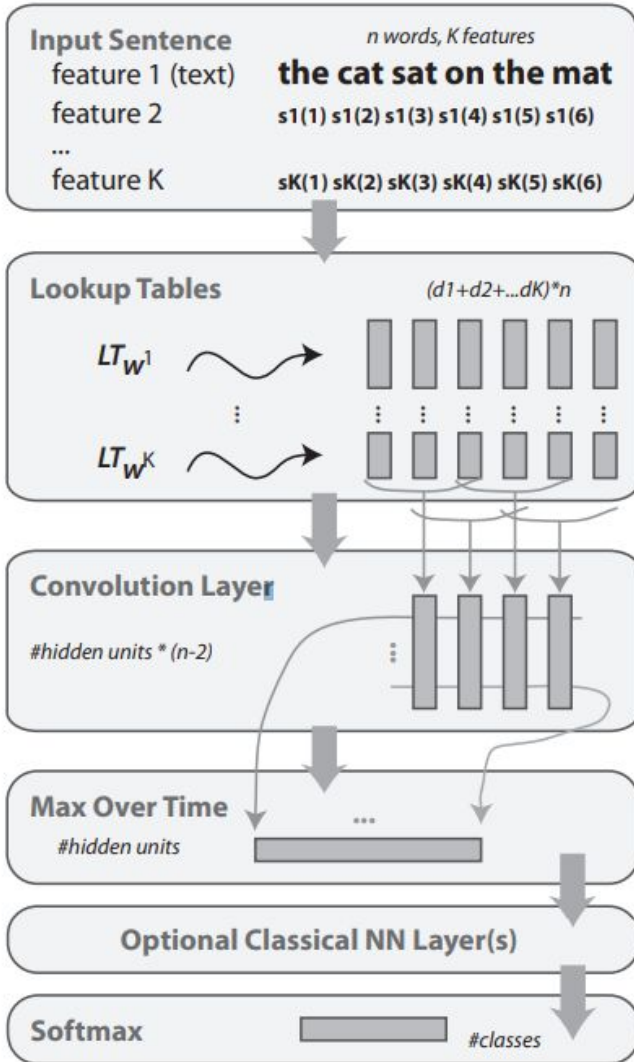
Mikolov: *word2vec*

2018

Collobert and Weston: *A unified architecture for natural language processing: Deep neural networks...*

XLNet  
RoBERTA

GPT3.5



# Modeling and Vector Semantics

... would be vowel or consonant.

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2003

... Blei et al.: *[ LDA Topic Modeling ]*

... Golov: *word2vec*

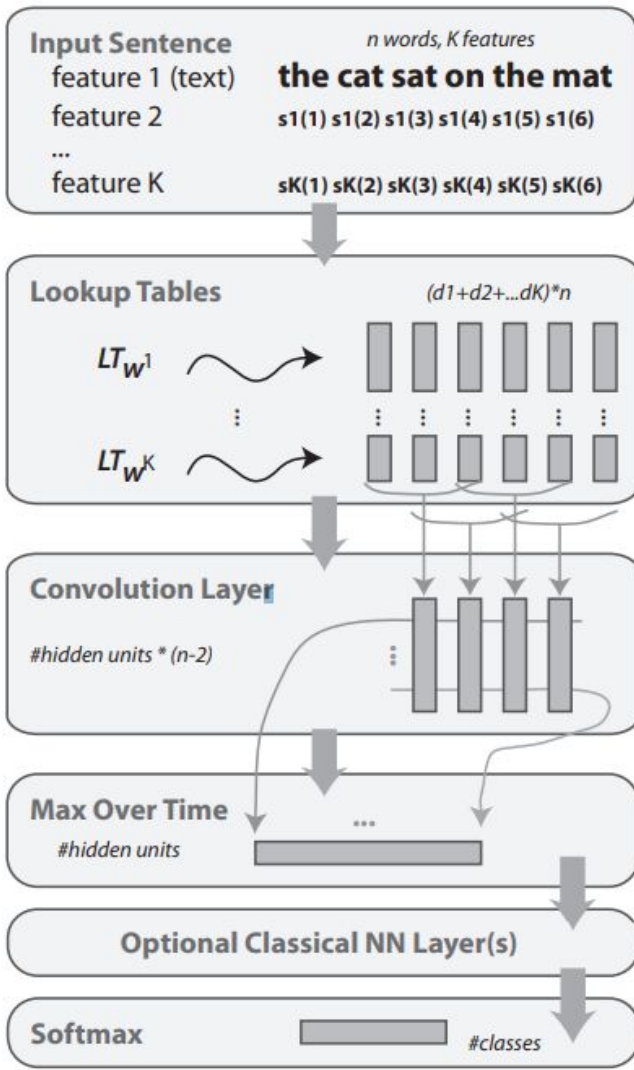
2018

POS, Chunking (Shallow Parsing), NER, SRL, Modified Language Modelling

... architecture for natural language processing: Deep neural networks...

XLNet  
RoBERTA

GPT3.5



1913

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M  
of



logarithmic scale

# Modeling and Vector Semantics

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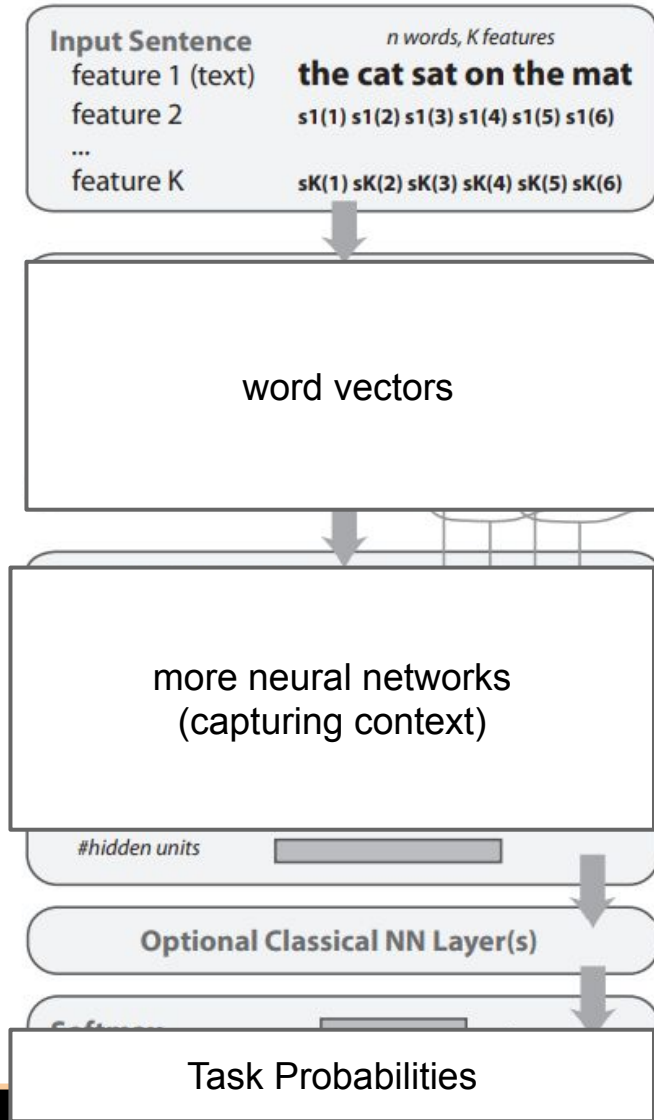
2018

POS, Chunking (Shallow Parsing), NER, SRL, Modified Language Modelling

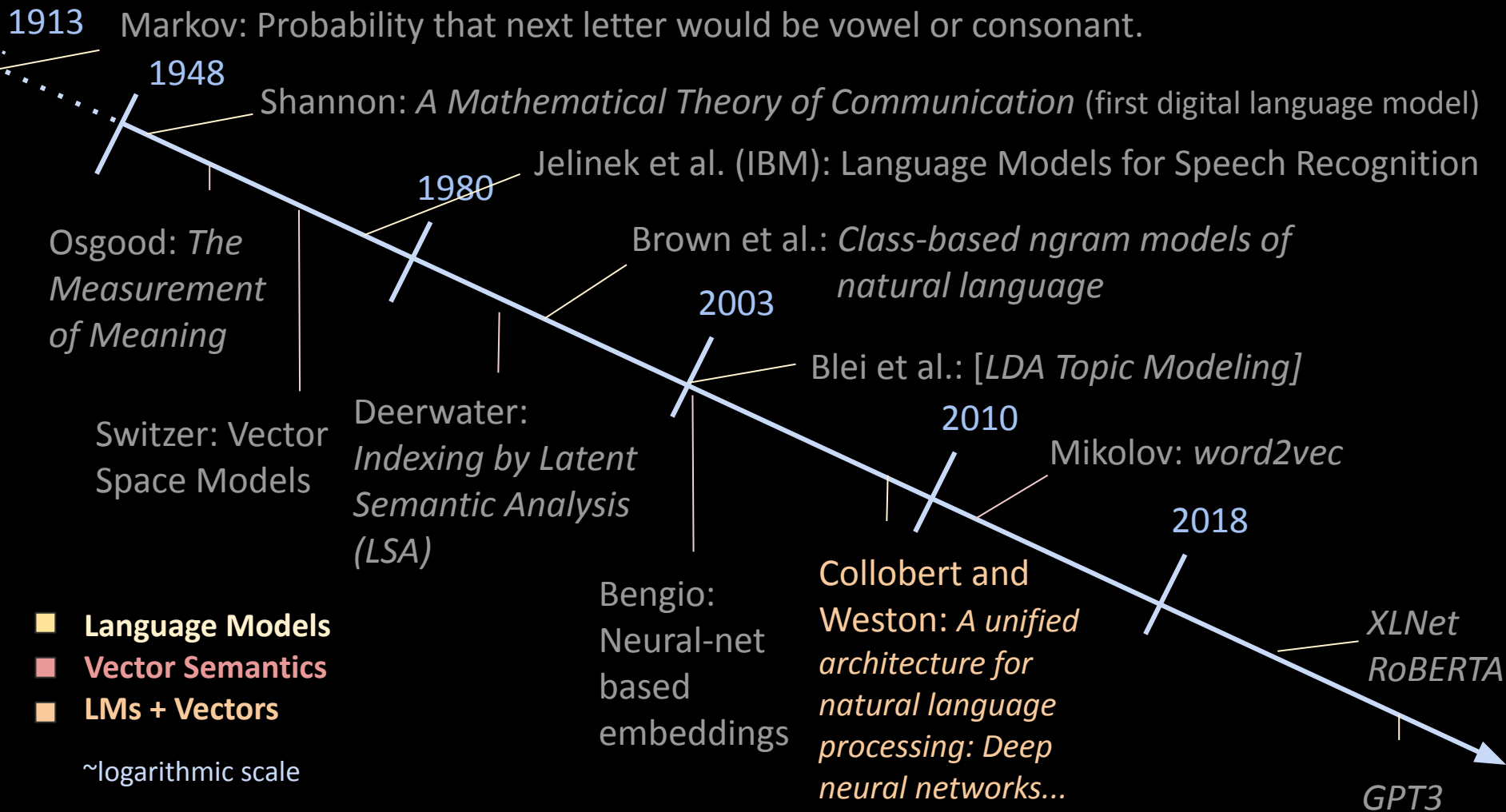
XLNet  
RoBERTA

architecture for natural language processing: Deep neural networks...

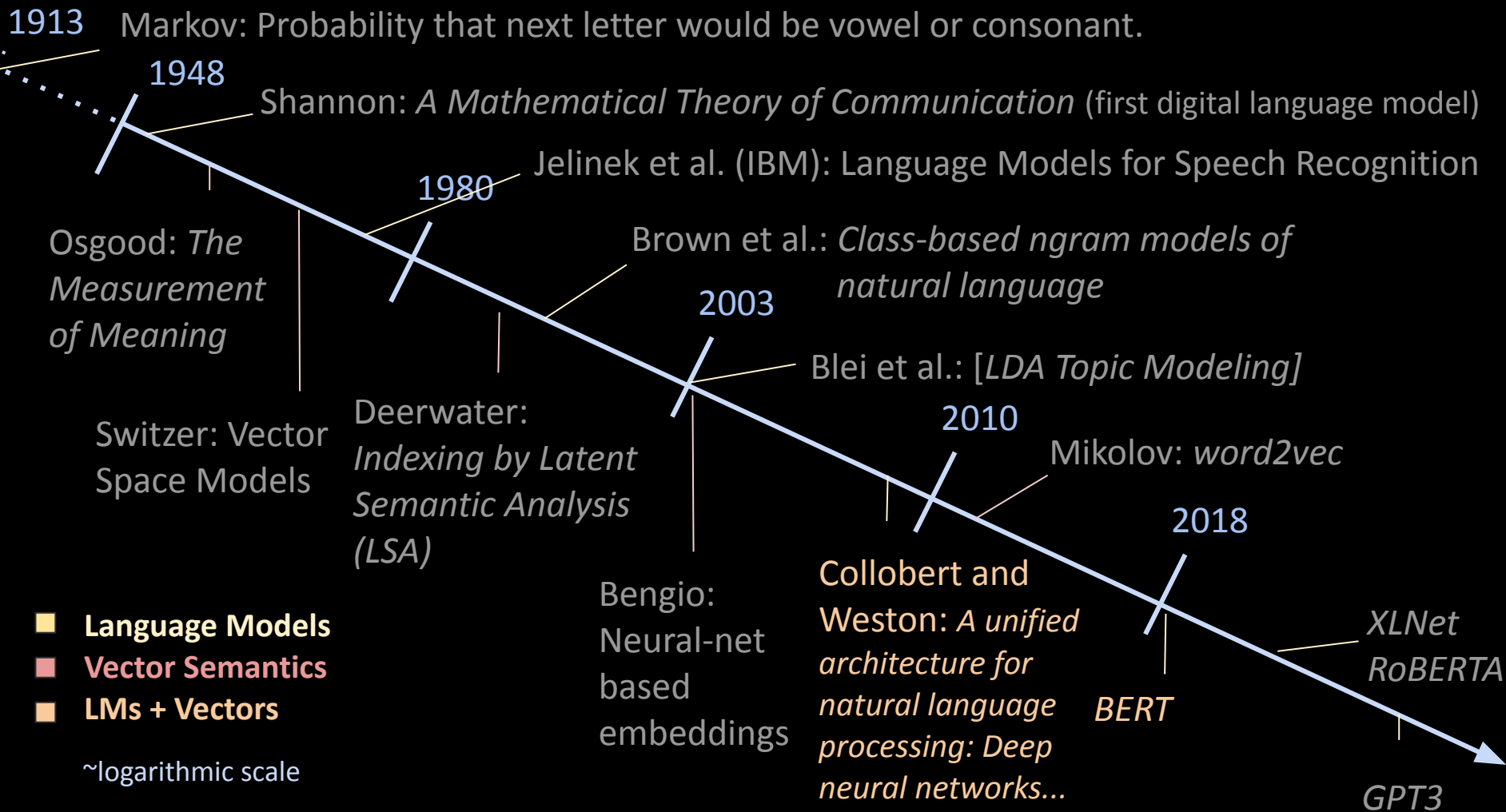
GPT3.5



# Timeline: *Language Modeling* and *Vector Semantics*



# Timeline: *Language Modeling* and *Vector Semantics*





# BERT: Pre-training of Deep Bidirectional Transformers for Language Understanding

Jacob Devlin Ming-Wei Chang Kenton Lee Kristina Toutanova  
Google AI Language  
{jacobdevlin, mingweichang, kentonl, kristout}@google.com

## Abstract

We introduce a new language representation model called **BERT**, which stands for **Bidirectional Encoder Representations from Transformers**. Unlike recent language representation models (Peters et al., 2018a; Radford et al., 2018), BERT is designed to pre-train deep bidirectional representations from unlabeled text by jointly conditioning on both left and right context in all layers. As a result, the pre-trained BERT model can be fine-tuned with just one additional output layer to create state-of-the-art models for a wide range of tasks, such as question answering and language inference, without substantial task-specific architecture modifications.

BERT is conceptually simple and empirically powerful. It obtains new state-of-the-art results on eleven natural language processing tasks, including pushing the GLUE score to 80.5% (7.7% point absolute improvement), MultiNLI accuracy to 86.7% (4.6% absolute improvement), SQuAD v1.1 question answering Test F1 to 93.2 (1.5 point absolute improvement) and SQuAD v2.0 Test F1 to 83.1 (5.1 point absolute improvement).

## 1 Introduction

Language model pre-training has been shown to be effective for improving many natural language processing tasks (Dai and Le, 2015; Peters et al., 2018a; Radford et al., 2018; Howard and Ruder, 2018). These include sentence-level tasks such as natural language inference (Bowman et al., 2015; Williams et al., 2018) and paraphrasing (Dolan and Brockett, 2005), which aim to predict the relationships between sentences by analyzing them holistically, as well as token-level tasks such as named entity recognition and question answering, where models are required to produce fine-grained output at the token level (Tjong Kim Sang and De Meulder, 2003; Rajpurkar et al., 2016).

There are two existing strategies for applying pre-trained language representations to downstream tasks: *feature-based* and *fine-tuning*. The feature-based approach, such as ELMo (Peters et al., 2018a), uses task-specific architectures that include the pre-trained representations as additional features. The fine-tuning approach, such as the Generative Pre-trained Transformer (OpenAI GPT) (Radford et al., 2018), introduces minimal task-specific parameters, and is trained on the downstream tasks by simply fine-tuning *all* pre-trained parameters. The two approaches share the same objective function during pre-training, where they use unidirectional language models to learn general language representations.

We argue that current techniques restrict the power of the pre-trained representations, especially for the fine-tuning approaches. The major limitation is that standard language models are unidirectional, and this limits the choice of architectures that can be used during pre-training. For example, in OpenAI GPT, the authors use a left-to-right architecture, where every token can only attend to previous tokens in the self-attention layers of the Transformer (Vaswani et al., 2017). Such restrictions are sub-optimal for sentence-level tasks, and could be very harmful when applying fine-tuning based approaches to token-level tasks such as question answering, where it is crucial to incorporate context from both directions.

In this paper, we improve the fine-tuning based approaches by proposing BERT: **Bidirectional Encoder Representations from Transformers**. BERT alleviates the previously mentioned unidirectionality constraint by using a “masked language model” (MLM) pre-training objective, inspired by the Cloze task (Taylor, 1953). The masked language model randomly masks some of the tokens from the input, and the objective is to predict the original vocabulary id of the masked

4171

Proceedings of NAACL-HLT 2019, pages 4171–4186

Minneapolis, Minnesota, June 2 - June 7, 2019. ©2019 Association for Computational Linguistics

## LMs + Vectors

~logarithmic scale

# Modeling and Vector Semantics

letter would be vowel or consonant.

Mathematical Theory of Communication (first digital language model)

Janek et al. (IBM): Language Models for Speech Recognition

Brown et al.: Class-based ngram models of natural language

2003

Blei et al.: [LDA Topic Modeling]

2010

Mikolov: word2vec

2018

Collobert and Weston: A unified architecture for natural language processing: Deep neural networks...

XLNet  
RoBERTA

BERT

GPT3

Bengio:  
Neural-net  
based  
embeddings



## BERT Rediscovered the Classical NLP Pipeline

Ian Tenney<sup>1</sup> Dipanjan Das<sup>1</sup> Ellie Pavlick<sup>1,2</sup>

<sup>1</sup>Google Research <sup>2</sup>Brown University

{iftenney, dipanjand, epavlick}@google.com

### Abstract

Pre-trained text encoders have rapidly advanced the state of the art on many NLP tasks. We focus on one such model, BERT, and aim to quantify where linguistic information is captured within the network. We find that the model represents the steps of the traditional NLP pipeline in an interpretable and localizable way, and that the regions responsible for each step appear in the expected sequence: POS tagging, parsing, NER, semantic roles, then coreference. Qualitative analysis reveals that the model can and often does adjust this pipeline dynamically, revising lower-level decisions on the basis of disambiguating information from higher-level representations.

relationships between sentences by analyzing them holistically, as well as token-level tasks such as named entity recognition and question answering, where models are required to produce fine-grained output at the token level (Tjong Kim Sang and De Meulder, 2003; Rajpurkar et al., 2016).

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of the network directly, to assess whether there exist localizable regions associated with distinct types of linguistic decisions. Such work has produced evidence that deep language models can encode a range of syntactic and semantic information (e.g. Shi et al., 2016; Belinkov, 2018; Tenney et al., 2019), and that more complex structures are represented hierarchically in the higher layers of the model (Peters et al., 2018b; Blevins et al., 2018).

We build on this latter line of work, focusing on the BERT model (Devlin et al., 2019), and use a suite of probing tasks (Tenney et al., 2019) derived from the traditional NLP pipeline to quantify where specific types of linguistic information are

Bengio:  
Neural-net  
based  
embeddings

## and *Vector Semantics*

owel or consonant.

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Language Models for Speech Recognition

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BERT

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GPT3

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LMs + Vectors

~logarithmic scale

## Abstract

We introduce a new language model called BERT, which is a Bidirectional Encoder Representations from Transformers. Unlike recent language generation models (Peters et al., 2018), BERT is designed to be trained on deep bidirectional representations of unlabeled text by jointly conditioning on both left and right context in all layers. We show that the pre-trained BERT model can be used to create state-of-the-art models for a wide range of tasks, such as question-answer matching, sentence classification, and natural language inference, without using any task-specific architecture modifications.

BERT is conceptually simple and powerful. It obtains new state-of-the-art results on eleven natural language tasks, including pushing the GLUE score to 80.5% (7.7% point absolute improvement), SQuAD v1.1 question-answer matching to 93.2 (1.5 point improvement) and SQuAD v2.0 to 83.1 (5.1 point absolute improvement).

## 1 Introduction

Language model pre-training has been effective for improving many processing tasks (Dai and Le, 2018a; Radford et al., 2018; Hosoya et al., 2018). These include sentence-level natural language inference (Bowling et al., 2018) and paragraph-level summarization (Williams et al., 2018) and paragraph-level summarization (Williams et al., 2018), which aim to capture relationships between sentences by holistically, as well as token-level named entity recognition and question-answer matching, where models are required to process output at the token level (Tjong and De Meulder, 2003; Rajpurkar et al., 2016).

## Journalism Quarterly

DEVOTED TO RESEARCH STUDIES IN THE FIELD OF MASS COMMUNICATIONS

FALL 1953

## "Cloze Procedure": A New Tool For Measuring Readability

BY WILSON L. TAYLOR\*

*Here is the first comprehensive statement of a research method and its theory which were introduced briefly during a workshop at the 1953 AEJ convention. Included are findings from three pilot studies and two experiments in which "cloze procedure" results are compared with those of two readability formulas.*

"CLOZE PROCEDURE" IS A NEW PSYCHOLOGICAL tool for measuring the effectiveness of communication. The method is straightforward; the data are easily quantifiable; the findings seem to stand up.

At the outset, this tool was looked on as a new approach to "readability." It was so used in three pilot studies and two experiments, the main findings of which are reported here.

\*The writer is particularly obligated to Prof. Charles E. Osgood, University of Illinois, and Melvin R. Marks, Personnel Research Section, A.G.O., Department of the Army, for instigating and assisting in the series of efforts that yielded the notion of "cloze procedure." Both are experimental psychologists. Among others who have advised, encouraged or otherwise aided are these of the University of Illinois: Prof. Lee J. Cronbach, educational psychologist and statistician; Dean Wilbur Schramm, Division of Communications; Prof. Charles E. Swanson, Institute of Communications Research, and George R. Klare, psychologist, both of whom have authored articles on readability; and several journalism teachers who lent their classes. Kaiimer E. Stordahl and Clifford M. Christensen, until recently research associates of the Institute, also contributed.

First, the results of the new method were repeatedly shown to conform with the results of the Flesch and Dale-Chall devices for estimating readability. Then the scope broadened, and cloze procedure was pitted against those standard formulas.

If future research substantiates the results so far, this tool seems likely to have a variety of applications, both theoretical and practical, in other fields involving communication functions.

## THE "CLOZE UNIT"

At the heart of the procedure is a functional unit of measurement tentatively dubbed a "cloze." It is pronounced like the verb "close" and is derived from "closure." The last term is one gestalt psychology applies to the human tendency to complete a familiar but not-quite-finished pattern—to "see" a broken circle as a whole one, for example, by mentally closing up the gaps.

## ng and Vector Semantics

e vowel or consonant.

of Communication (first digital language model)

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l.: Class-based ngram models of natural language

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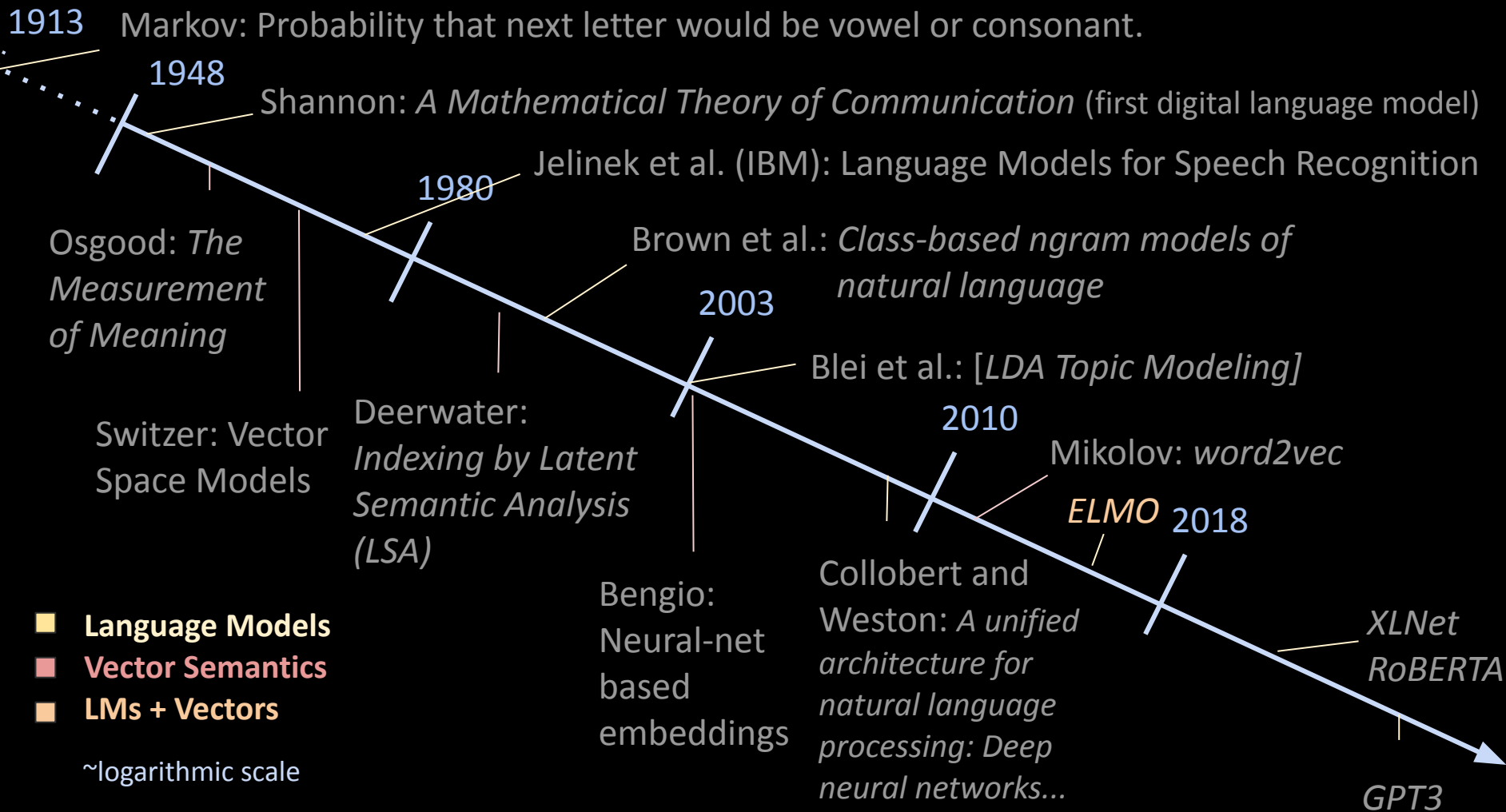
Collobert and Weston: A unified architecture for natural language processing: Deep neural networks...

XLNet  
RoBERTA

BERT

GPT3

# Timeline: *Language Modeling* and *Vector Semantics*



# Masked Language Modeling

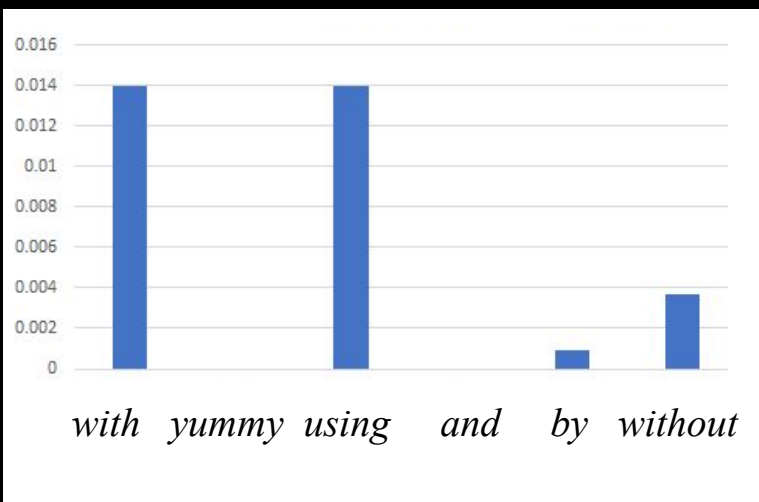
*Task: Estimate  $P(w_i | w_1, \dots, w_{i-1}, w_{i+1}, \dots, w_n)$*

*:P(masked word given history)*

*$P(\text{with} | \text{He ate the cake } \langle M \rangle \text{ the fork}) = ?$*

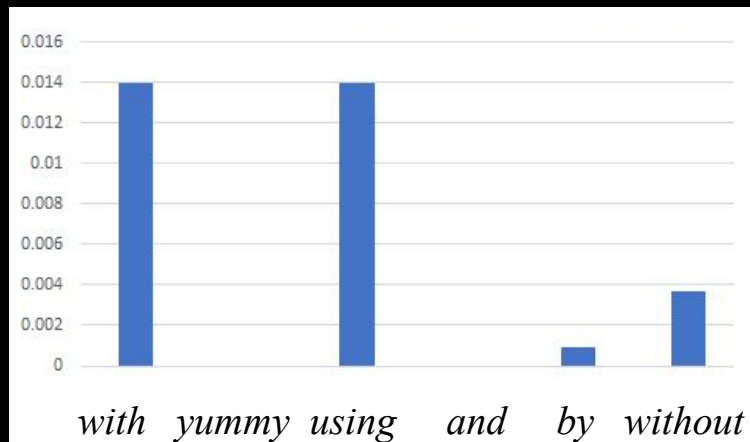
# Masked Language Modeling

*Task: Estimate  $P(w_i | w_1, \dots, w_{i-1}, w_{i+1}, \dots, w_n)$   
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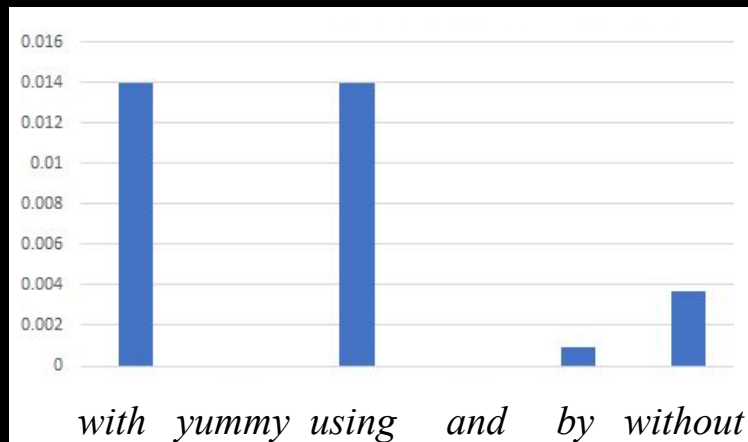
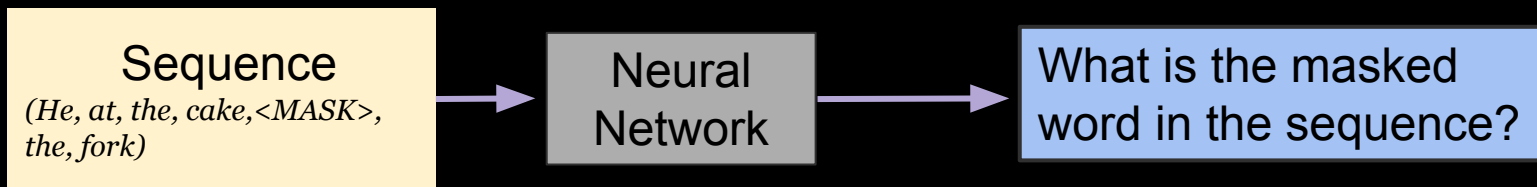
# Masked Language Modeling

*Task: Estimate  $P(w_i | w_1, \dots, w_{i-1}, w_{i+1}, \dots, w_n)$   
:P(masked word given history)  
 $P(\text{with} | \text{He ate the cake } \langle M \rangle \text{ the fork}) = ?$*



# Masked Language Modeling

*Task: Estimate  $P(w_i | w_1, \dots, w_{i-1}, w_{i+1}, \dots, w_n)$   
:P(masked word given history)  
 $P(\text{with} | \text{He ate the cake } \langle M \rangle \text{ the fork}) = ?$*



# Masked Language Modelling with DNN

He

ate

the

cake

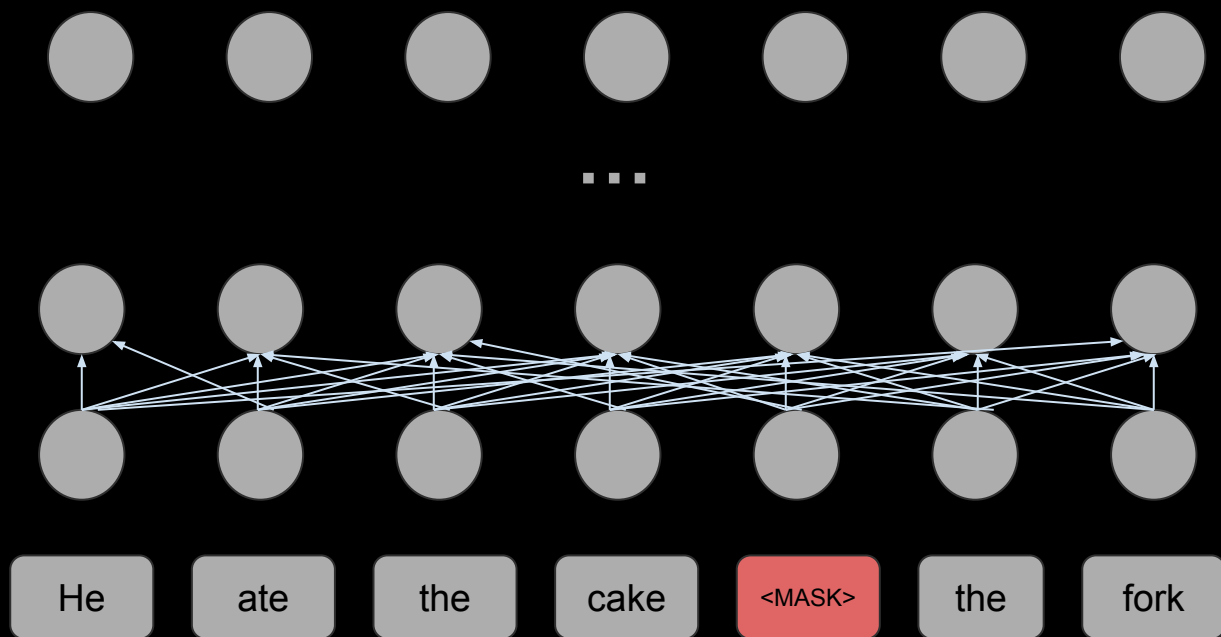
<MASK>

the

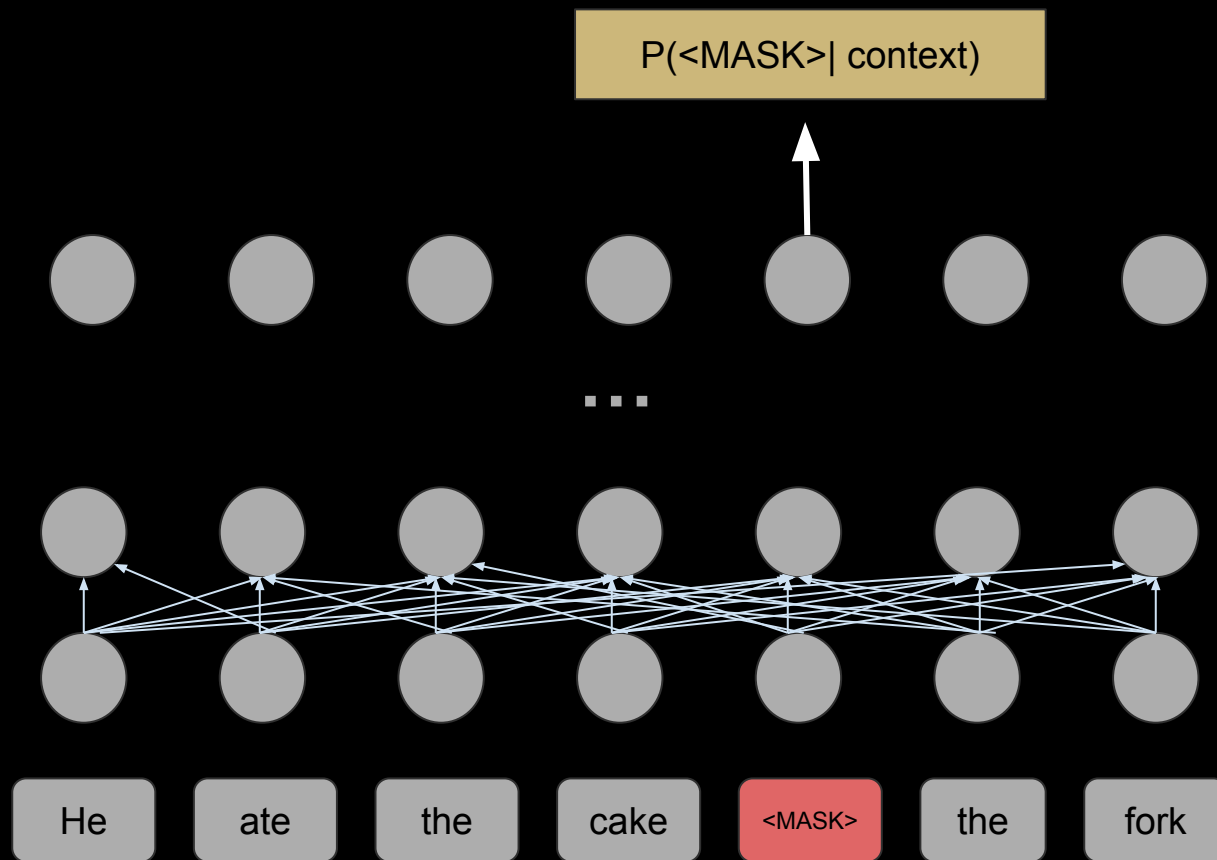
fork



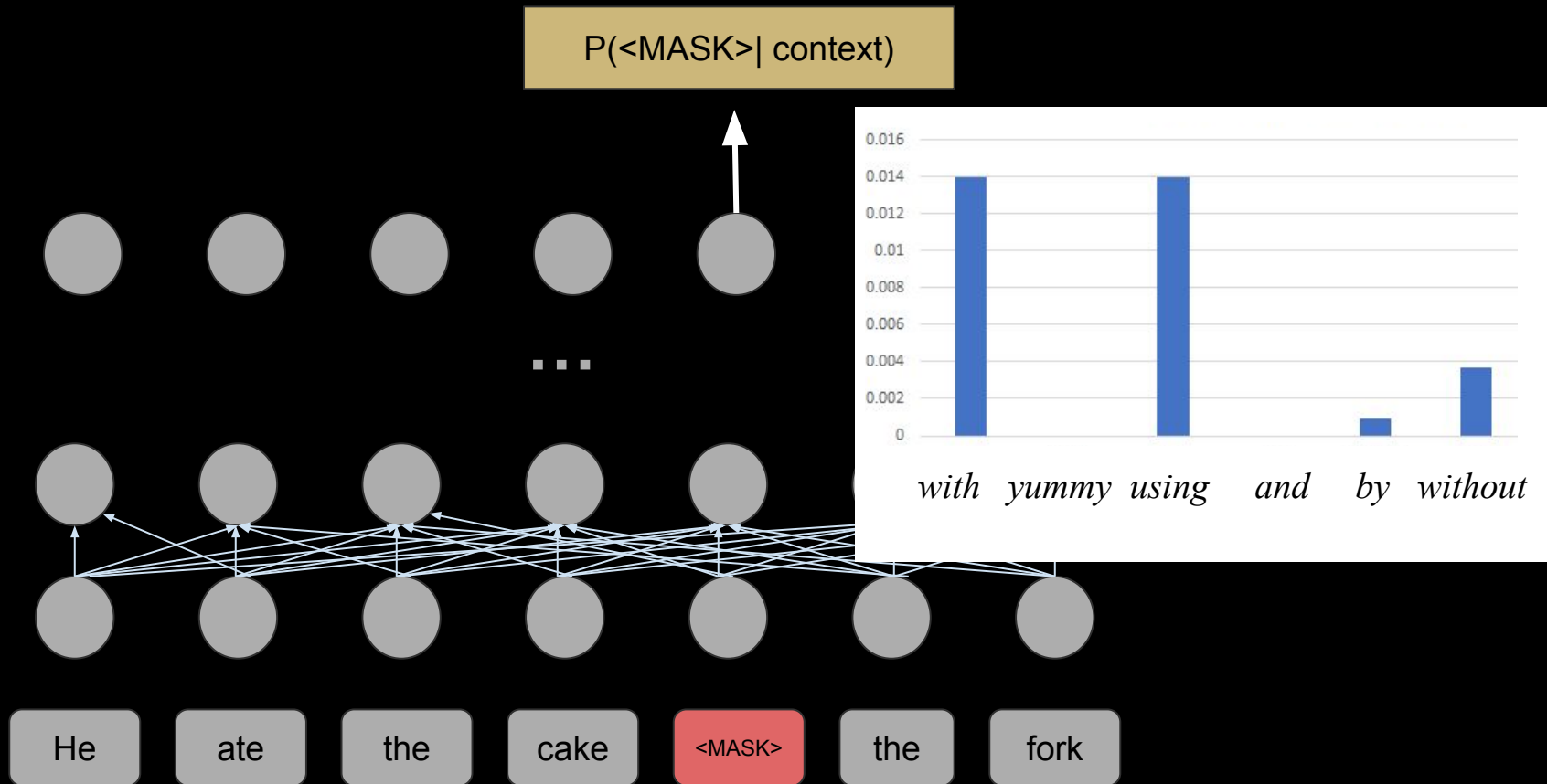
# Masked Language Modelling with DNN



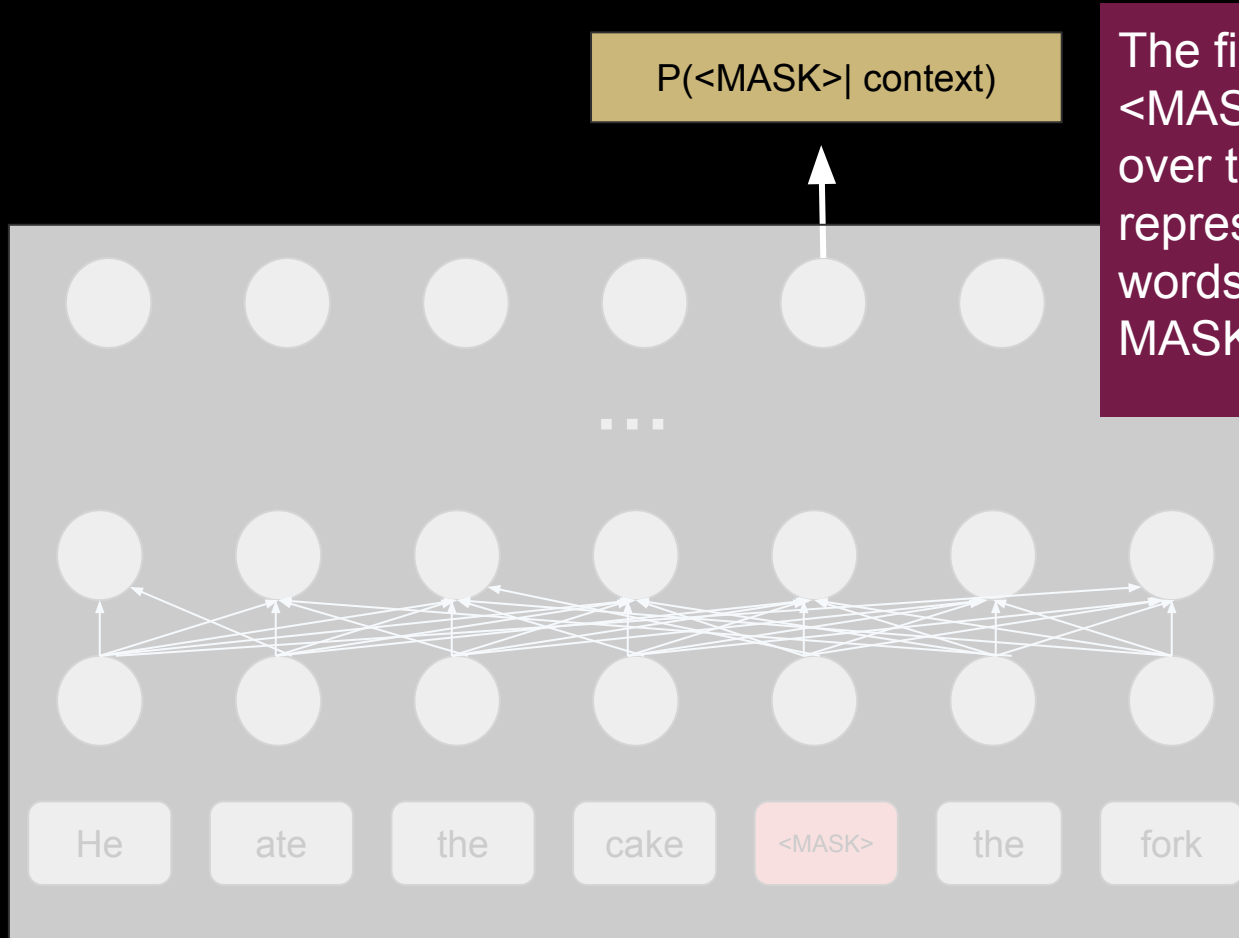
# Masked Language Modelling with DNN



# Masked Language Modelling with DNN

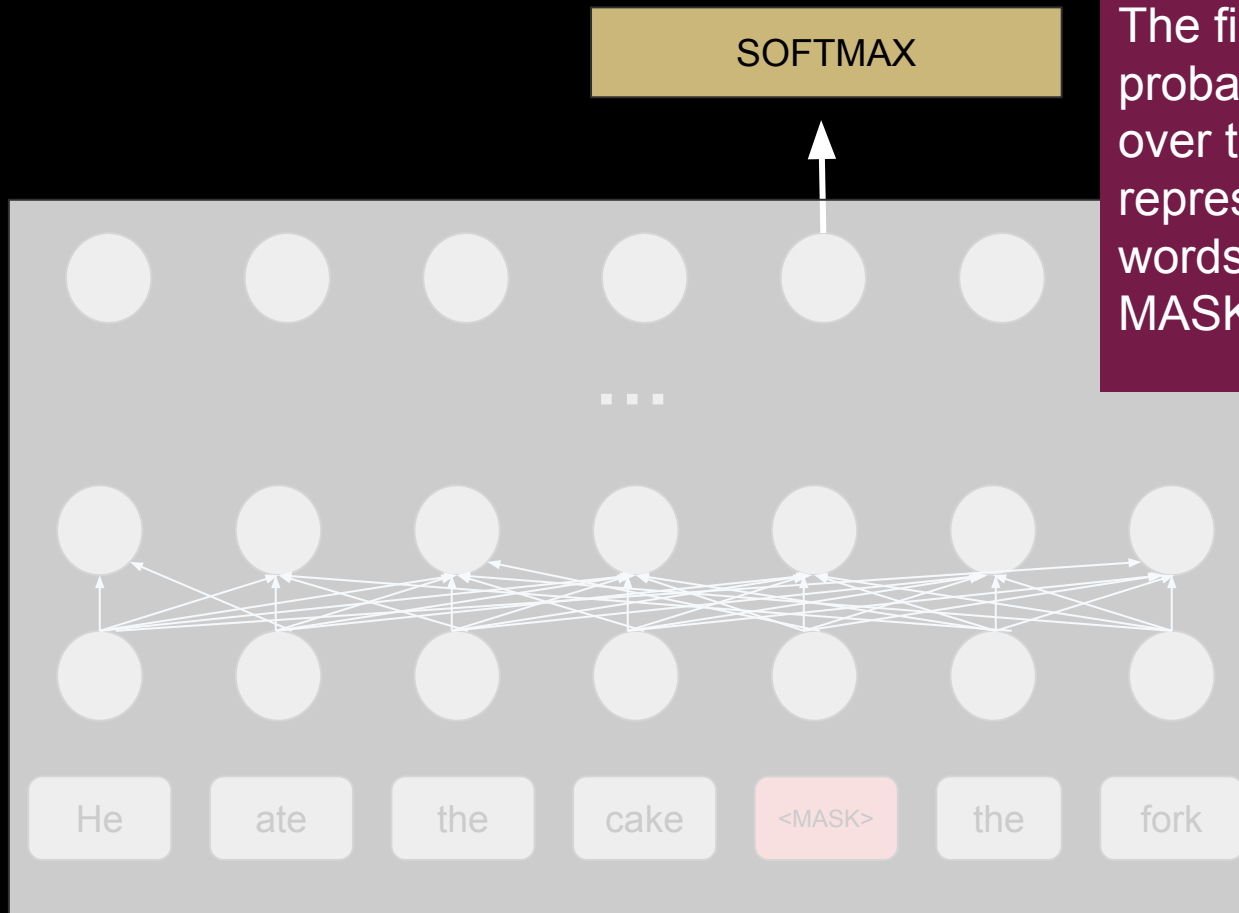


# Masked Language Modelling with ANN



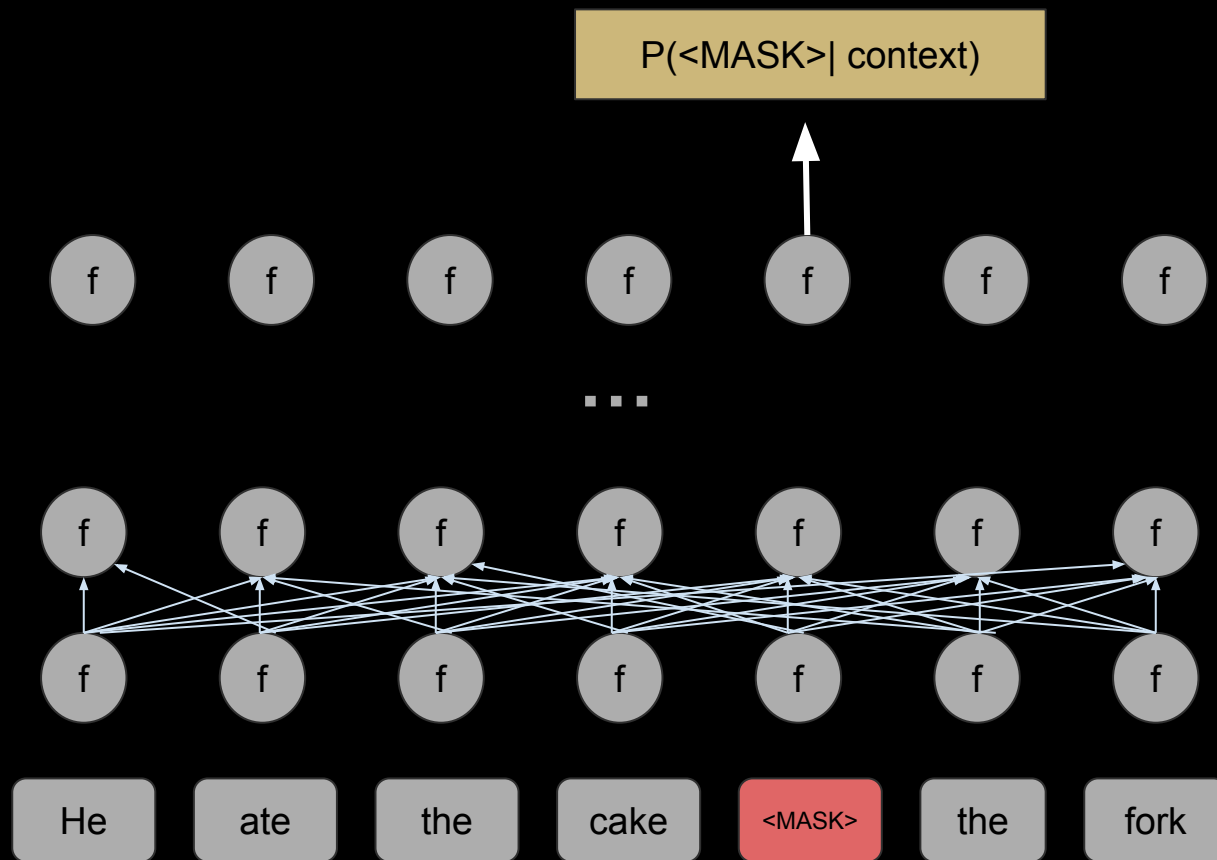
The final layer produces a  $<MASK>$  distribution over the vocabulary, representing the likely words to fill in the MASK-ed token

# Masked Language Modelling with DNN

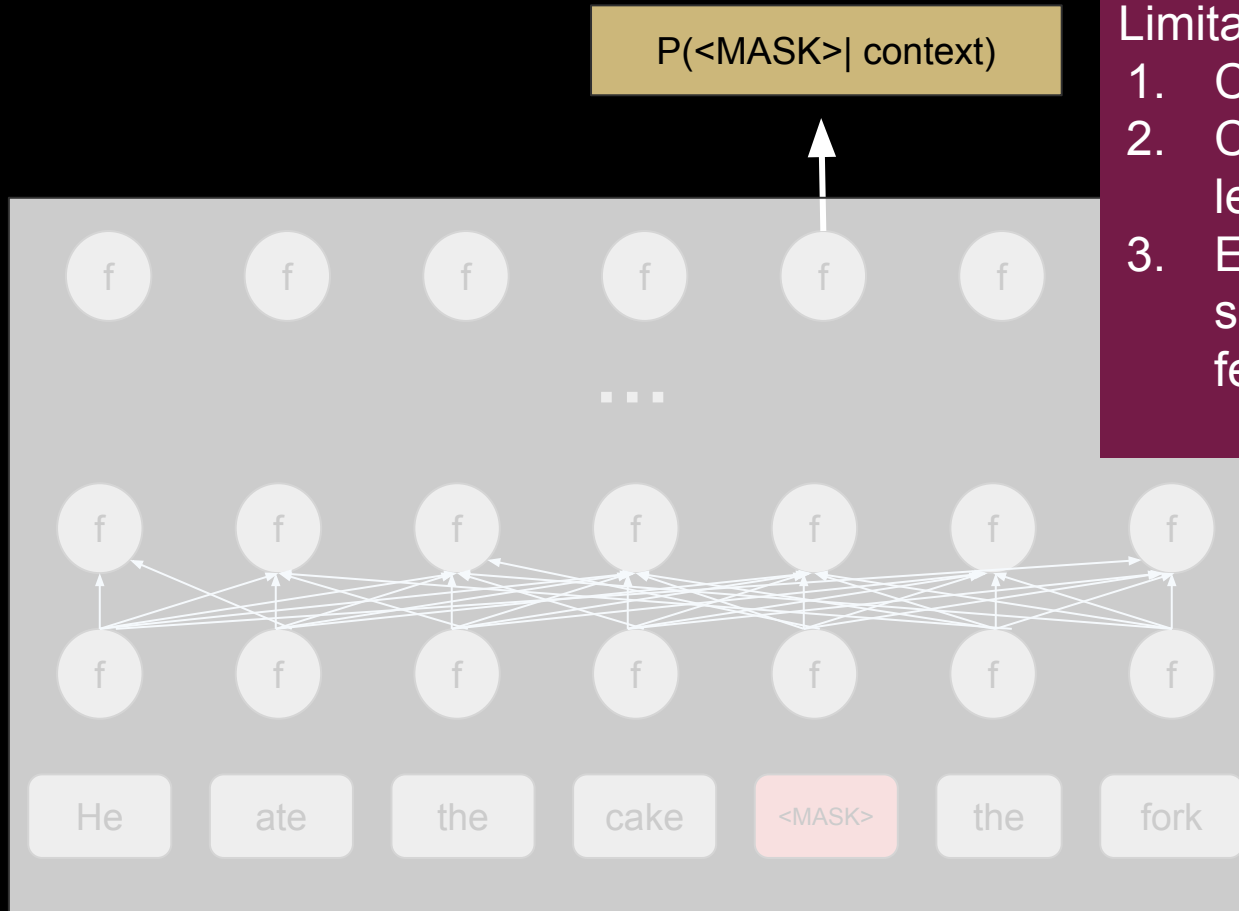


The final layer produces a probability distribution over the vocabulary, representing the likely words to fill in the MASK-ed token

# Masked Language Modelling with DNN



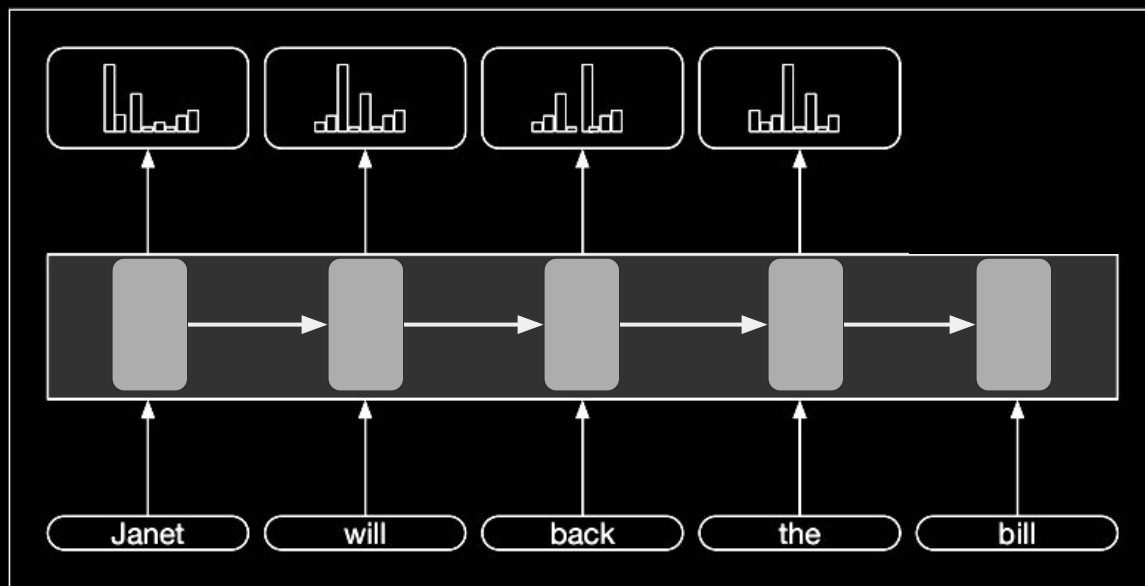
# Masked Language Modelling with DNN



## Limitations:

1. Can't handle order
2. Can't handle variable length sequences
3. Each parameter is specific to the input feature (token)

# Recurrent Neural Network



**Masked Language modeling  
with an RNN**



# Example: Forward Pass



(Geron, 2017)

```
#define forward pass graph:
```

```
 $h_{(0)} = 0$ 
```

```
for i in range(1, len(x)):
```

```
     $h_{(i)} = g(U h_{(i-1)} + W x_{(i)})$  #update hidden state
```

```
     $y_{(i)} = f(V h_{(i)})$  #update output
```

# Example: Forward Pass



```
#define forward pass graph:
```

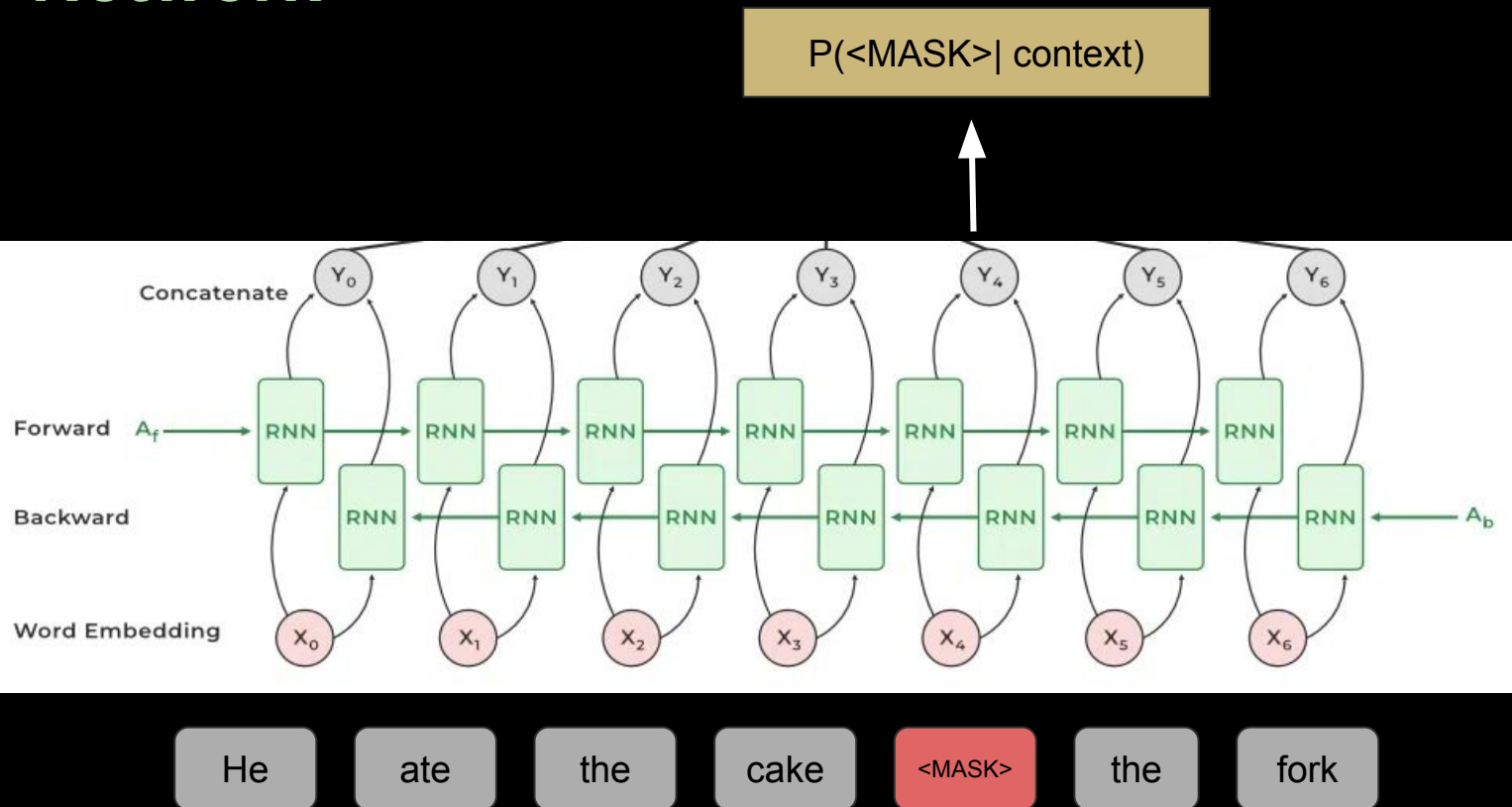
```
 $h_{(0)} = 0$ 
```

```
for i in range(1, len(x)):
```

```
     $h_{(i)} = \tanh(\text{matmul}(U, h_{(i-1)}) + \text{matmul}(W, x_{(i)}))$  #update hidden state
```

```
     $y_{(i)} = \text{softmax}(\text{matmul}(V, h_{(i)}))$  #update output
```

# Masked Language Modelling with Recurrent Network



# Vanishing/exploding gradients (Computational graph)

GRU and LSTM cells solve.

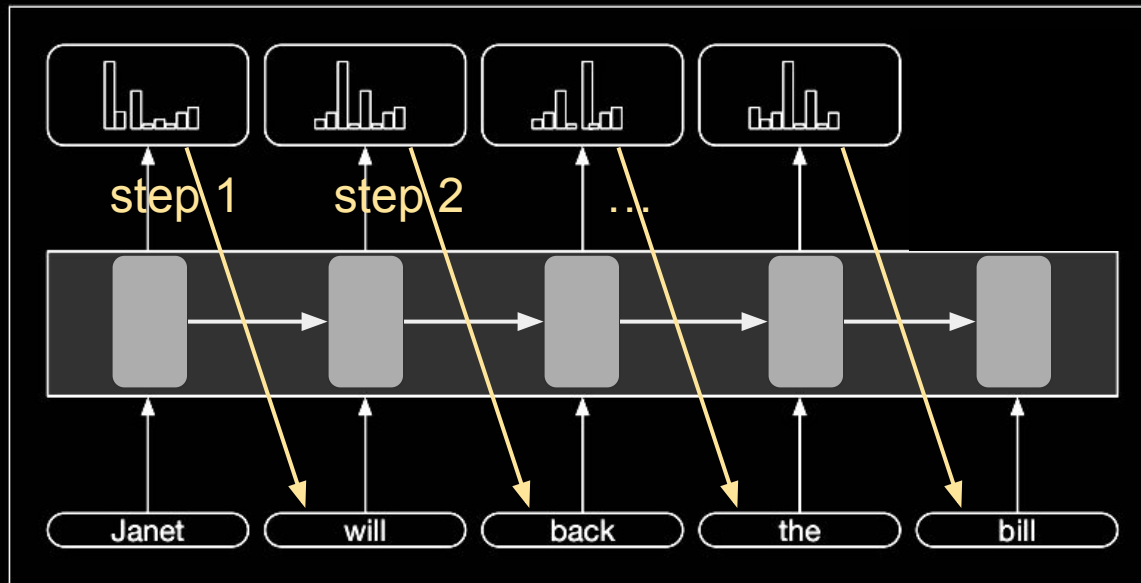
# RNN Limitation:

## Losing Track of Long Distance Dependencies

*The horse which was raced past the barn tripped.*



# RNN: Limitation: Not parallelizable



Language modeling  
with an RNN

# Next Lecture

- Deep dive into Self Attention (Vaswani et al., 2017)
- Masked Language Modelling using Transformers (Devlin et al., 2019)

# Part 2: Transformer and Self-attention

Nikita Soni

[nisoni@cs.stonybrook.edu](mailto:nisoni@cs.stonybrook.edu)

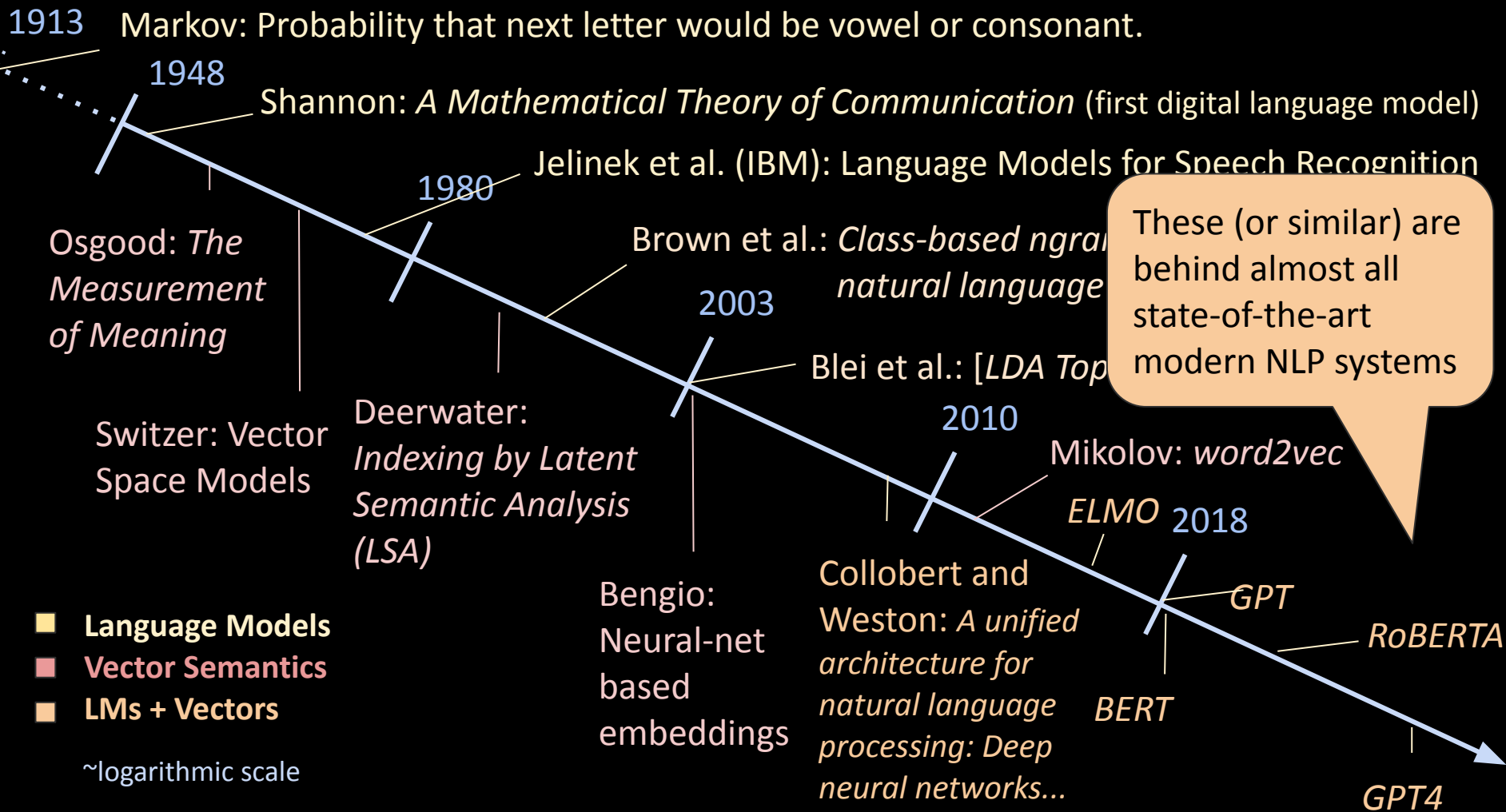
CSE538 - Spring 2024



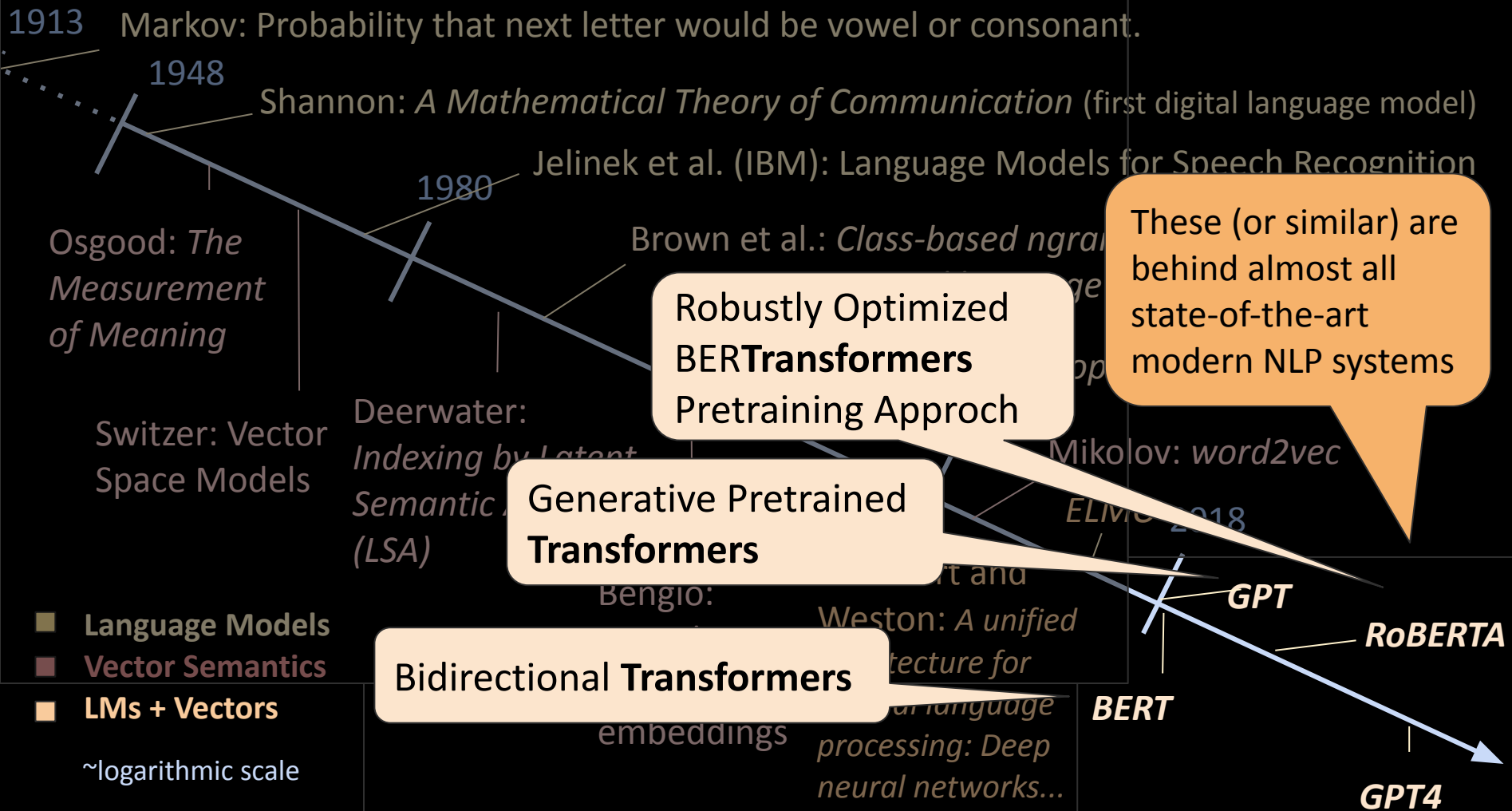
# Recap: RNN Limitations

- Difficult to capture long-distance dependencies
- Not parallelizable -- need sequential processing.
  - Slow computation for long sequences
- Vanishing or exploding gradients

# Timeline: *Language Modeling* and *Vector Semantics*



# Timeline: *Language Modeling* and *Vector Semantics*



# The Transformer: Motivation

- Capture long-distance dependencies
- Preserving sequential distances / periodicity
- Capture multiple relationships
- Easy to parallelize -- don't need sequential processing.

# Introducing the Transformer

## Attention Is All You Need

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nikip@google.com

**Jakob Uszkoreit\***  
Google Research  
usz@google.com

**Llion Jones\***  
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llion@google.com

**Aidan N. Gomez\* †**  
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illia.polosukhin@gmail.com

### Abstract

The dominant sequence transduction models are based on complex recurrent or convolutional neural networks that include an encoder and a decoder. The best performing models also connect the encoder and decoder through an attention mechanism. We propose a new simple network architecture, the Transformer, based solely on attention mechanisms, dispensing with recurrence and convolutions entirely. Experiments on two machine translation tasks show these models to be superior in quality while being more parallelizable and requiring significantly less time to train. Our model achieves 28.4 BLEU on the WMT 2014 English-to-German translation task, improving over the existing best results, including ensembles, by over 2 BLEU. On the WMT 2014 English-to-French translation task, our model establishes a new single-model state-of-the-art BLEU score of 41.0 after training for 3.5 days on eight GPUs, a small fraction of the training costs of the best models from the literature.

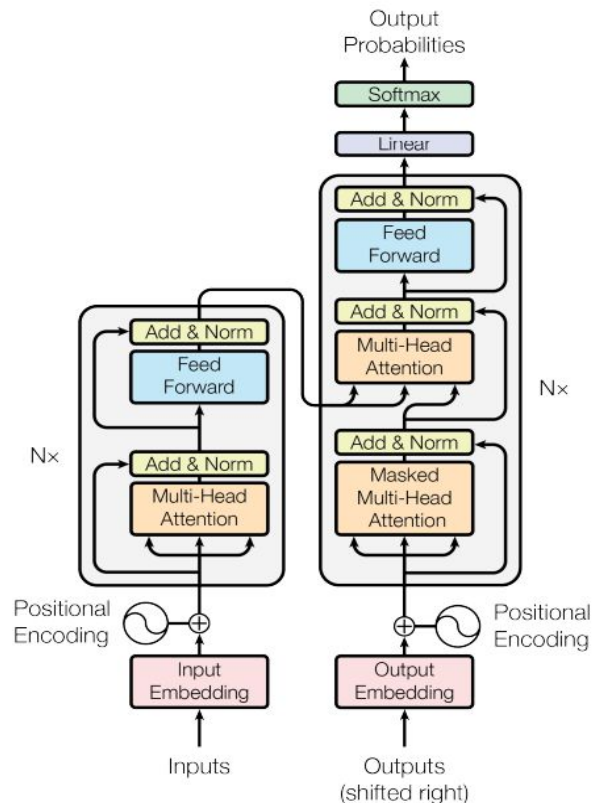


Figure 1: The Transformer - model architecture.

# Introducing the Transformer

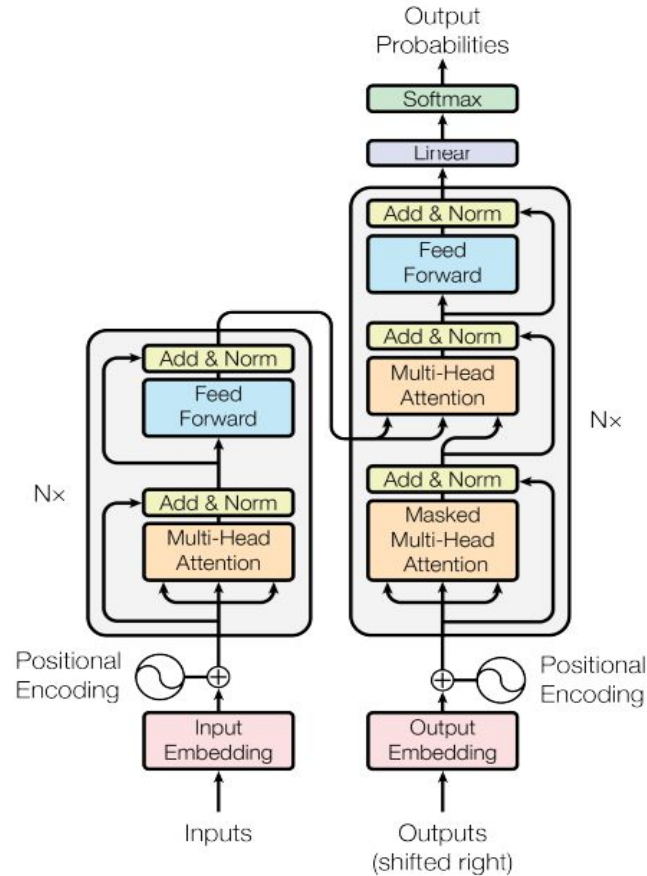
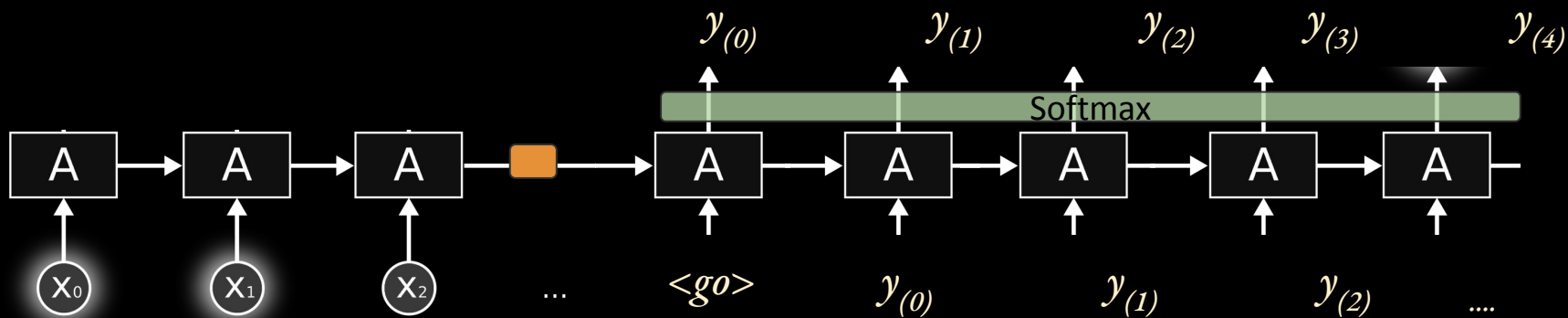
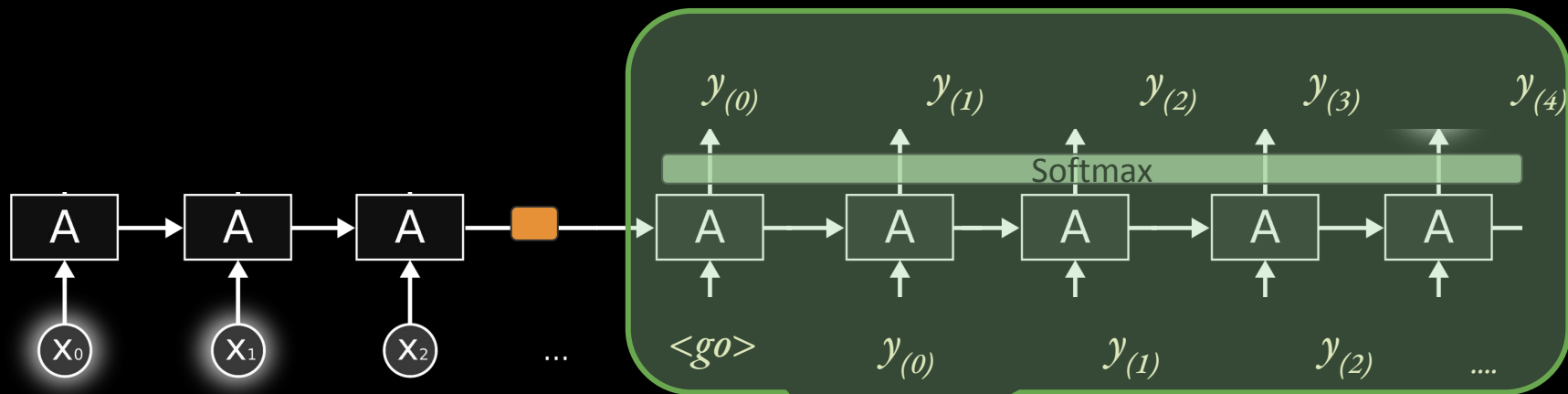


Figure 1: The Transformer - model architecture.

# Encoder-Decoder (Simpler Representation)



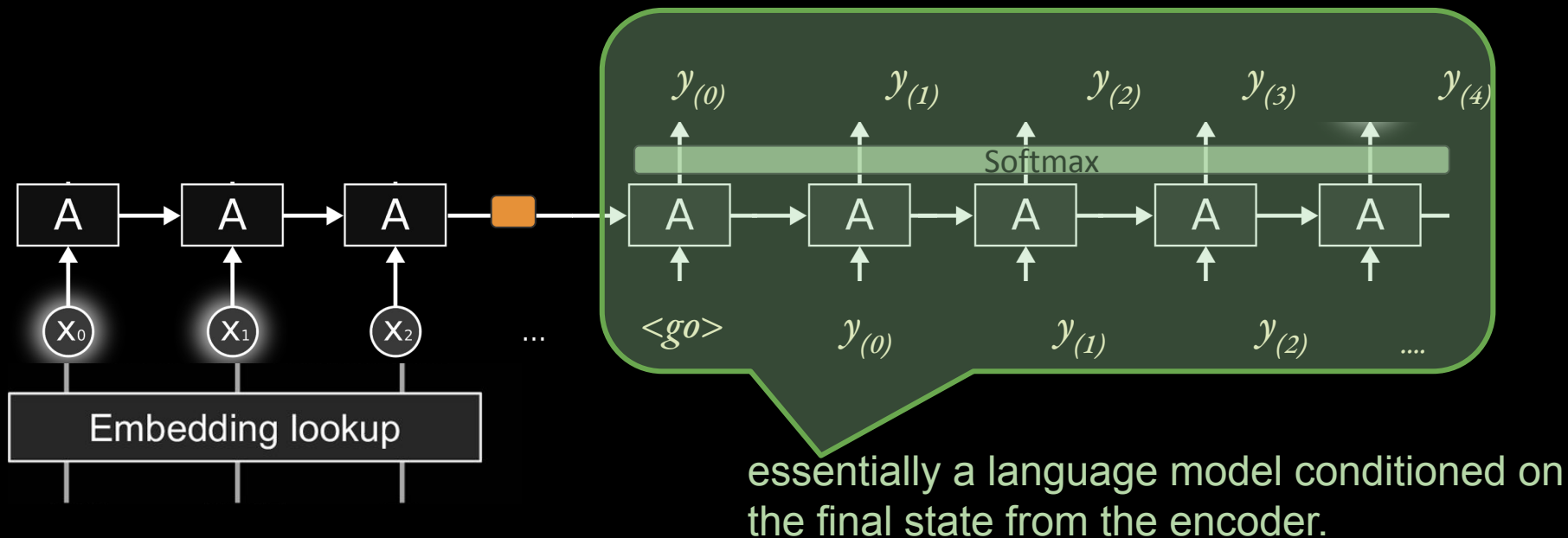
# Encoder-Decoder (Simpler Representation)



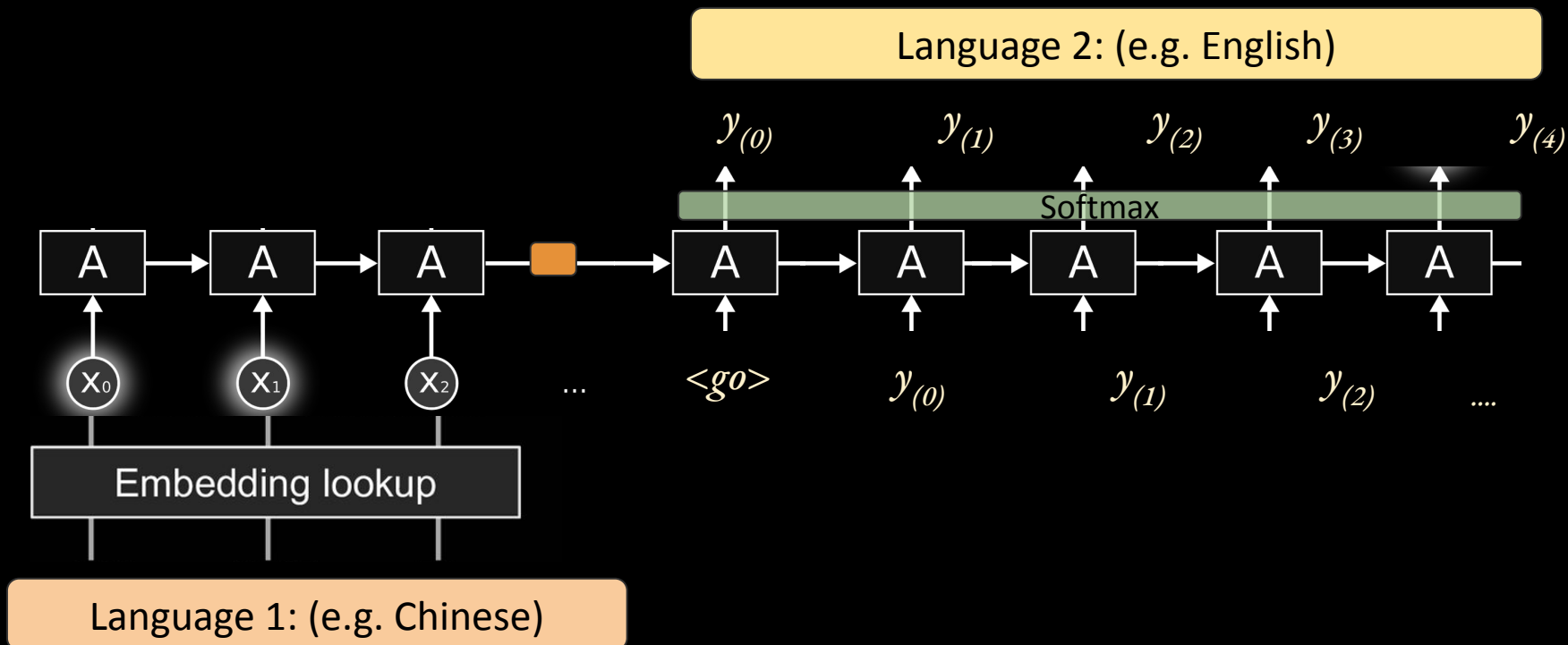
essentially a language model conditioned on the final state from the encoder.



# Encoder-Decoder (Simpler Representation)



# Encoder-Decoder (Simpler Representation)

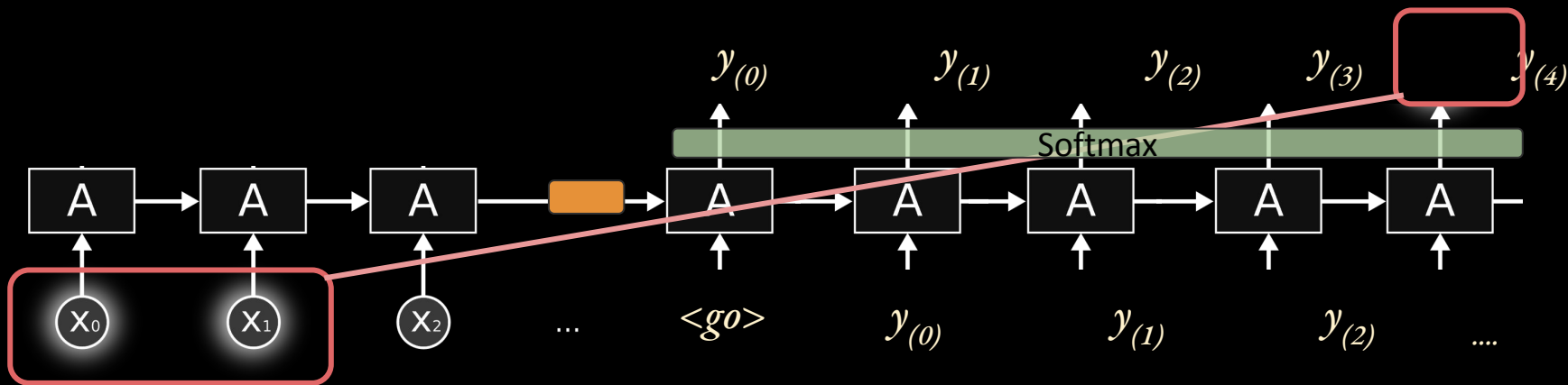


# Encoder-Decoder

Challenge:

*The ball was kicked by kayla.*

- Long distance dependency when translating:



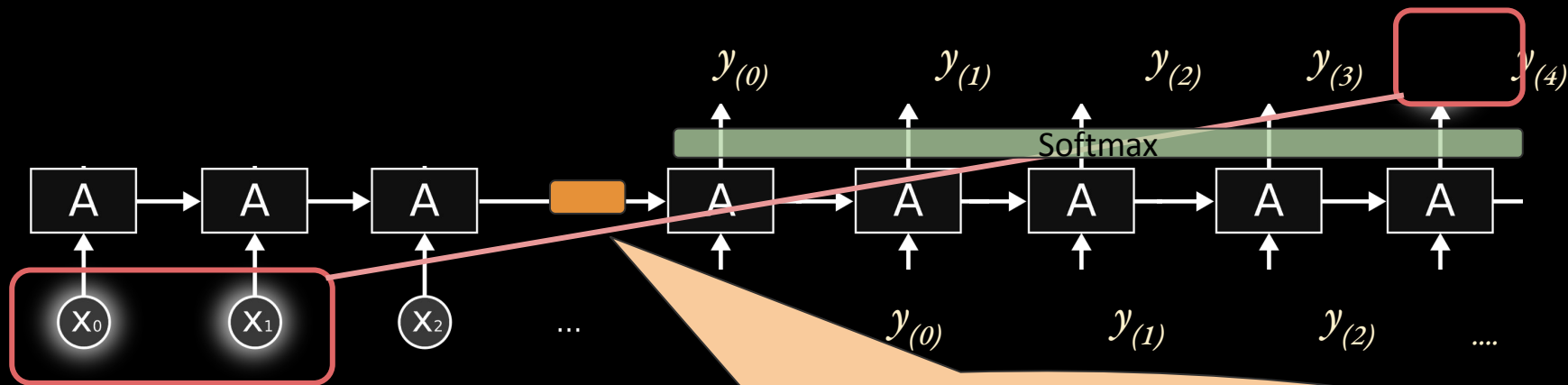
*Kayla kicked the ball.*

# Encoder-Decoder

Challenge:

*The ball was kicked by kayla.*

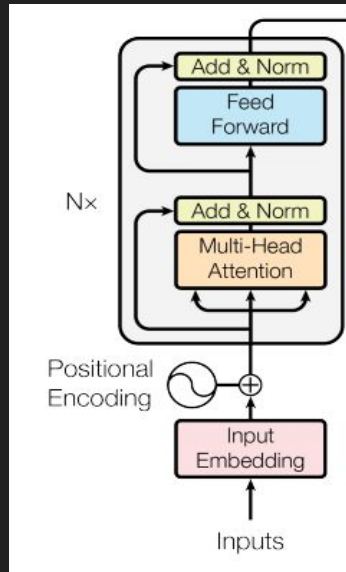
- Long distance dependency when translating:



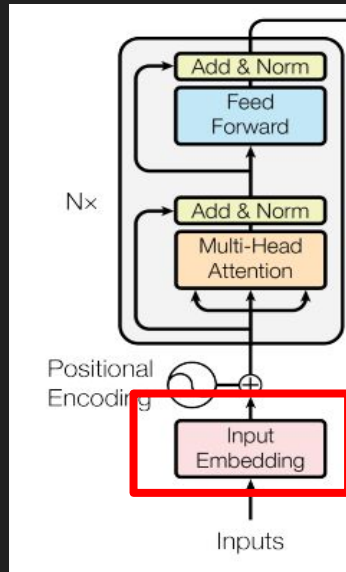
*Kayla kicked the ball.*

A lot of responsibility put fixed-size hidden state passed from encoder to decoder

# Encoder



# Encoder: Input Embedding



# Input Embedding

Original Sentence

Tokenization

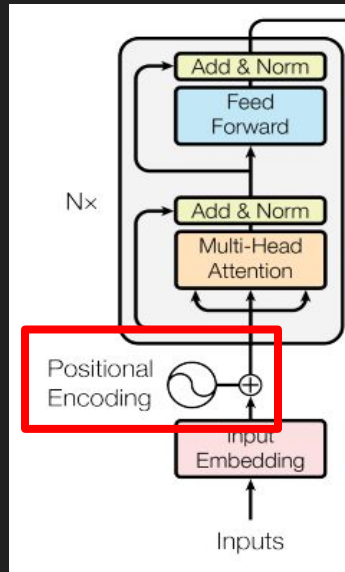
Input IDs

(embedding lookup: position in the vocab -  
FIXED)

Embeddings

(vector of size  $d_{\text{model}} = 512$  or  $1024$  or ...  
LEARNED)

# Encoder: Positional Encoding





# Positional Encoding

Original Sentence

(tokens)

Embeddings

(vector of size  $d_{\text{model}} = 512$  or  $1024$  or ...

Learned)

Positional Embedding

(vector of size  $d_{\text{model}} = 512$  or  $1024$  or ...

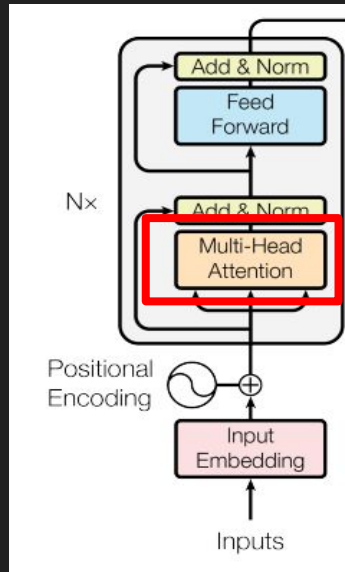
Can be Learned or Fixed)

# Positional Encoding

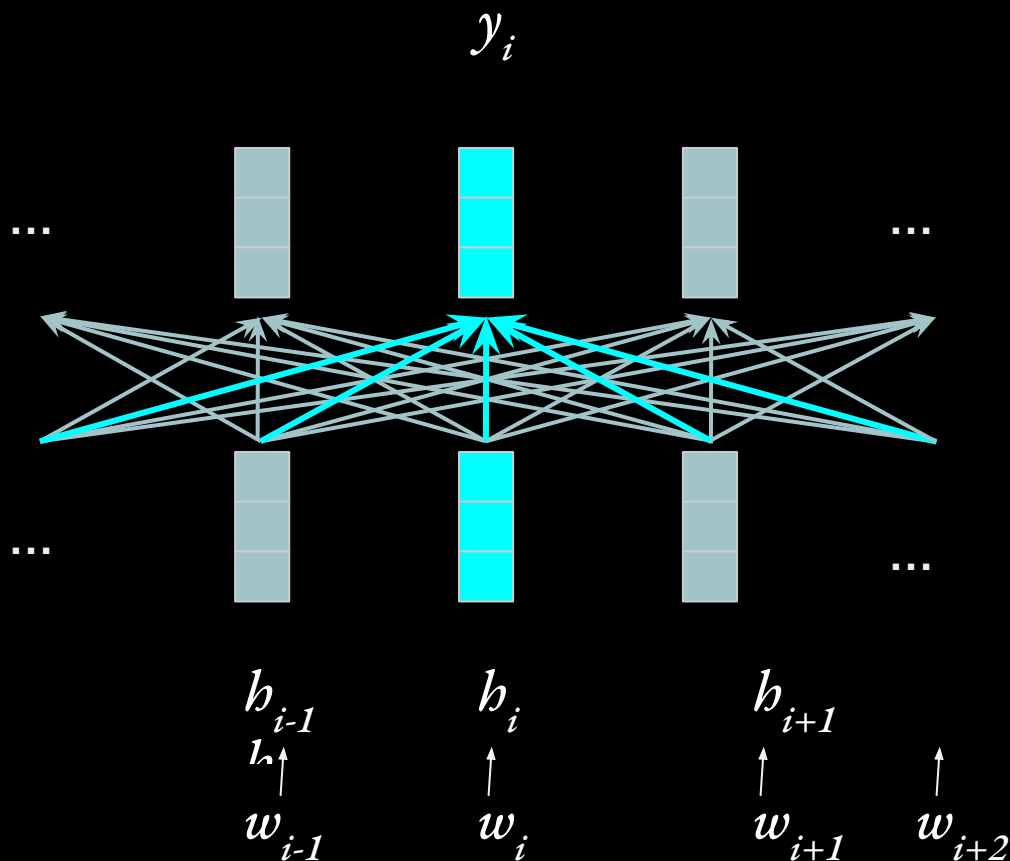
$$PE_{(pos, 2i)} = \sin(pos/10000^{2i/d_{\text{model}}})$$

$$PE_{(pos, 2i+1)} = \cos(pos/10000^{2i/d_{\text{model}}})$$

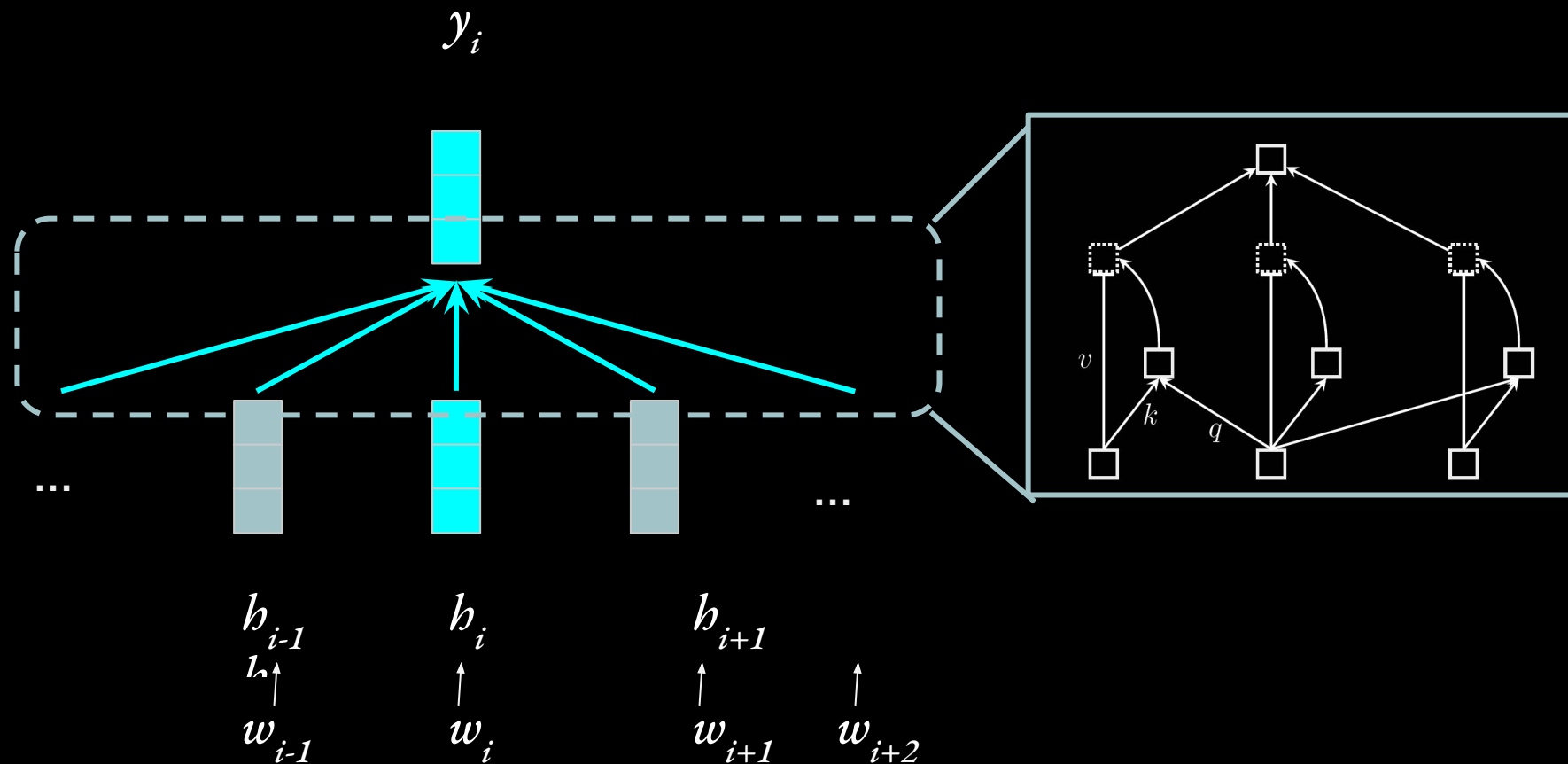
# Encoder: Multi-Head Attention



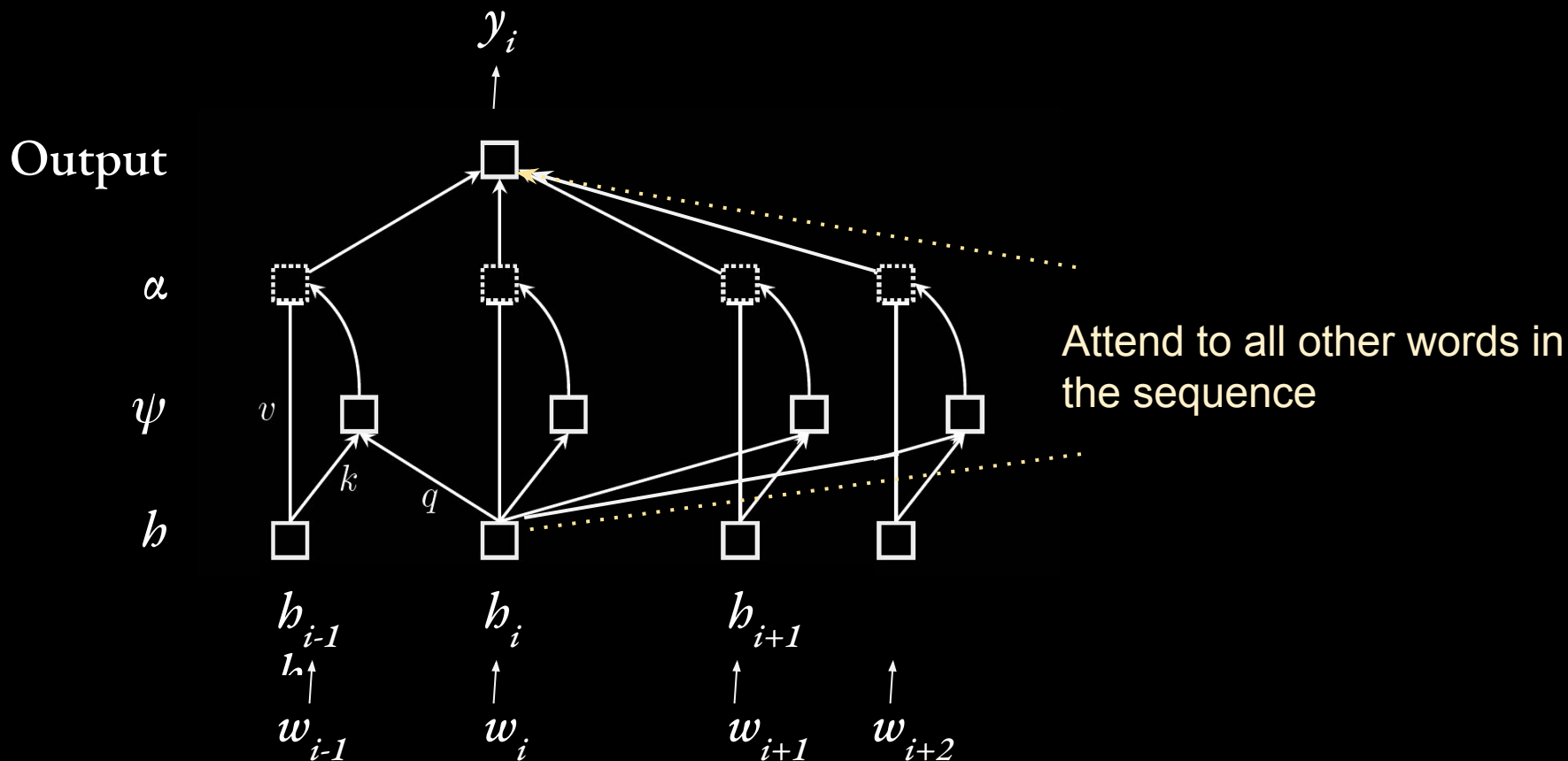
# The Transformer's Heart: Self-Attention



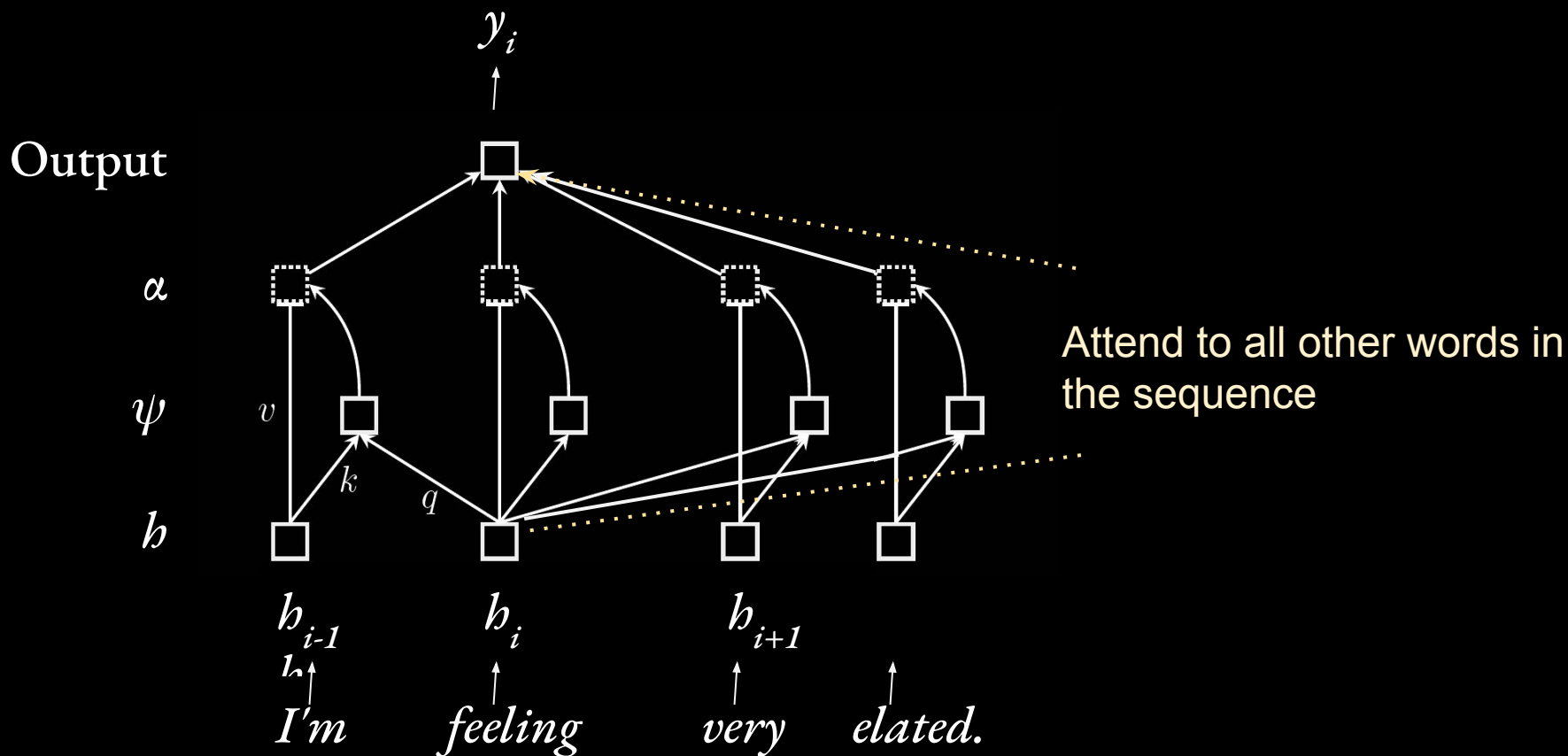
# The Transformer's Heart: Self-Attention



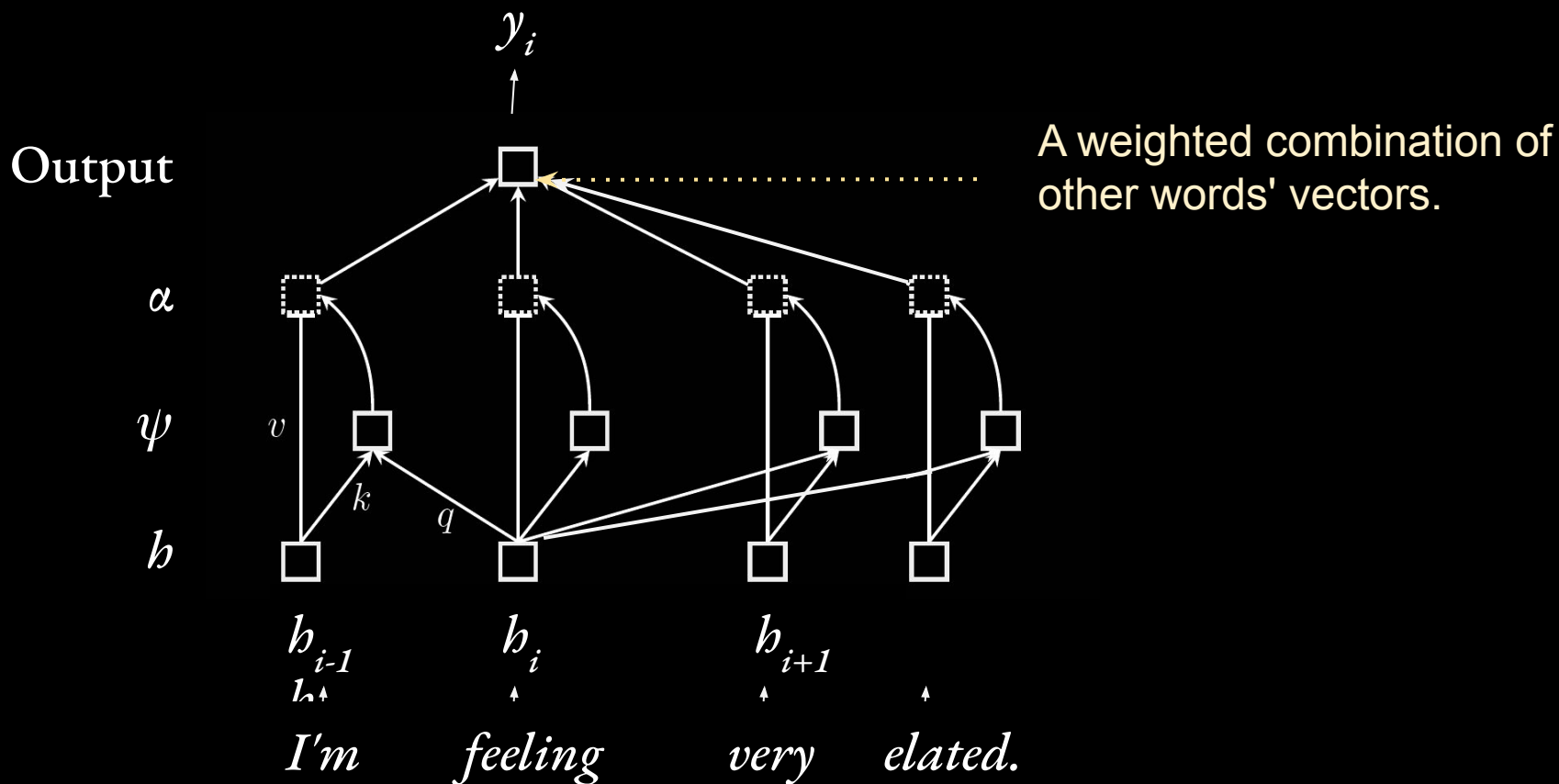
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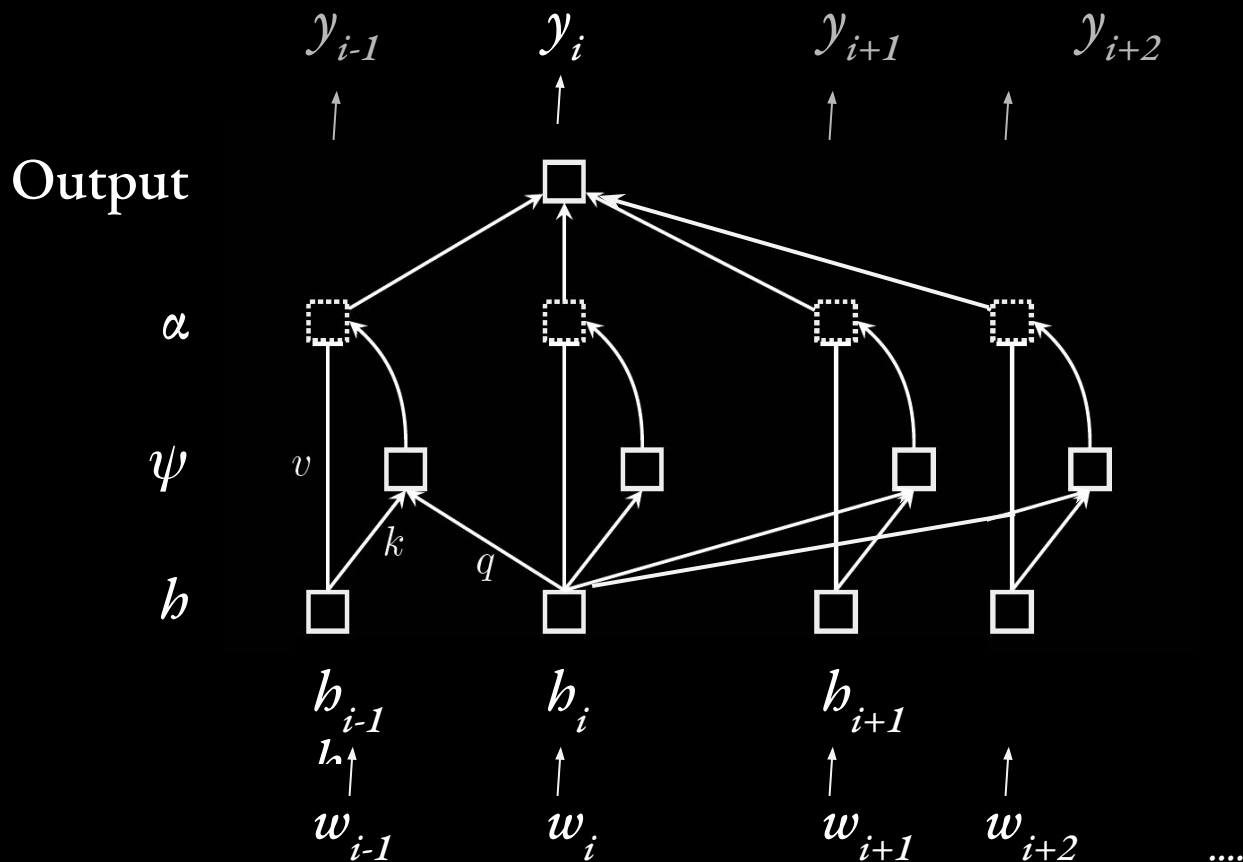


# The Transformer's Heart: Self-Attention

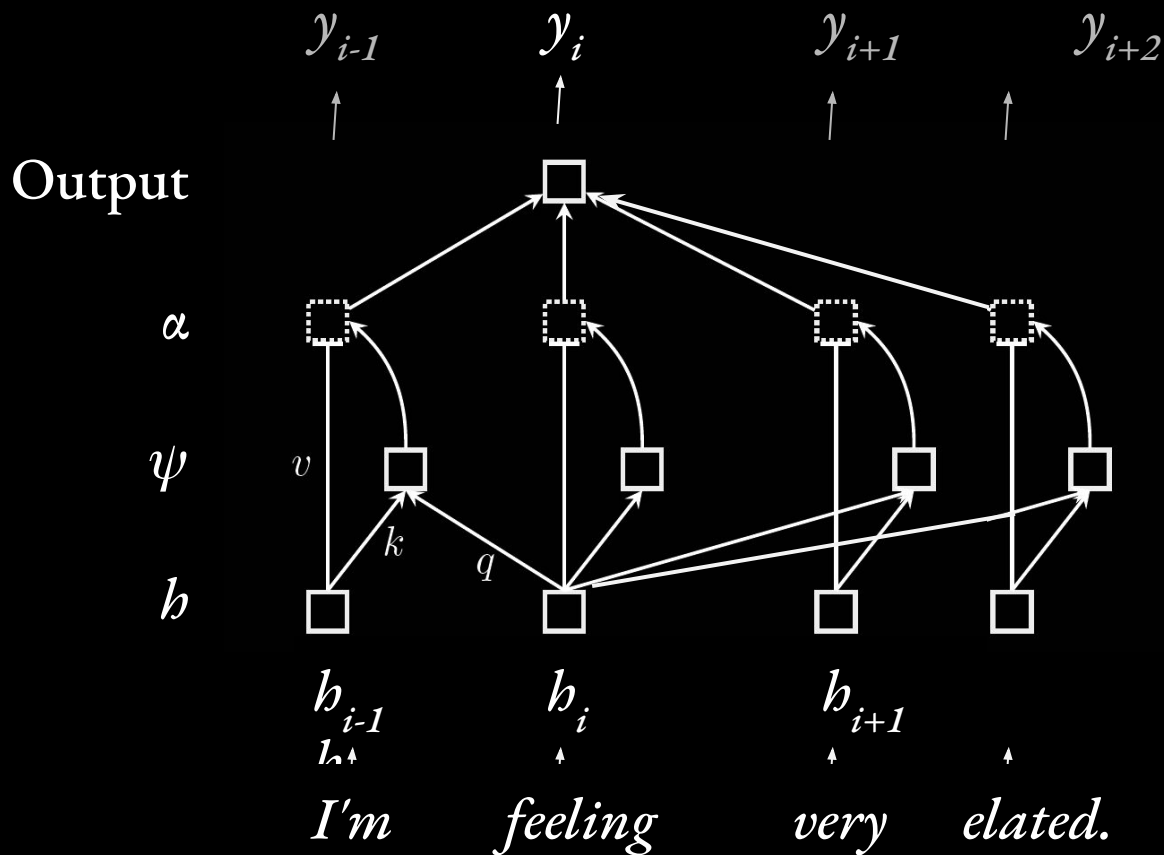




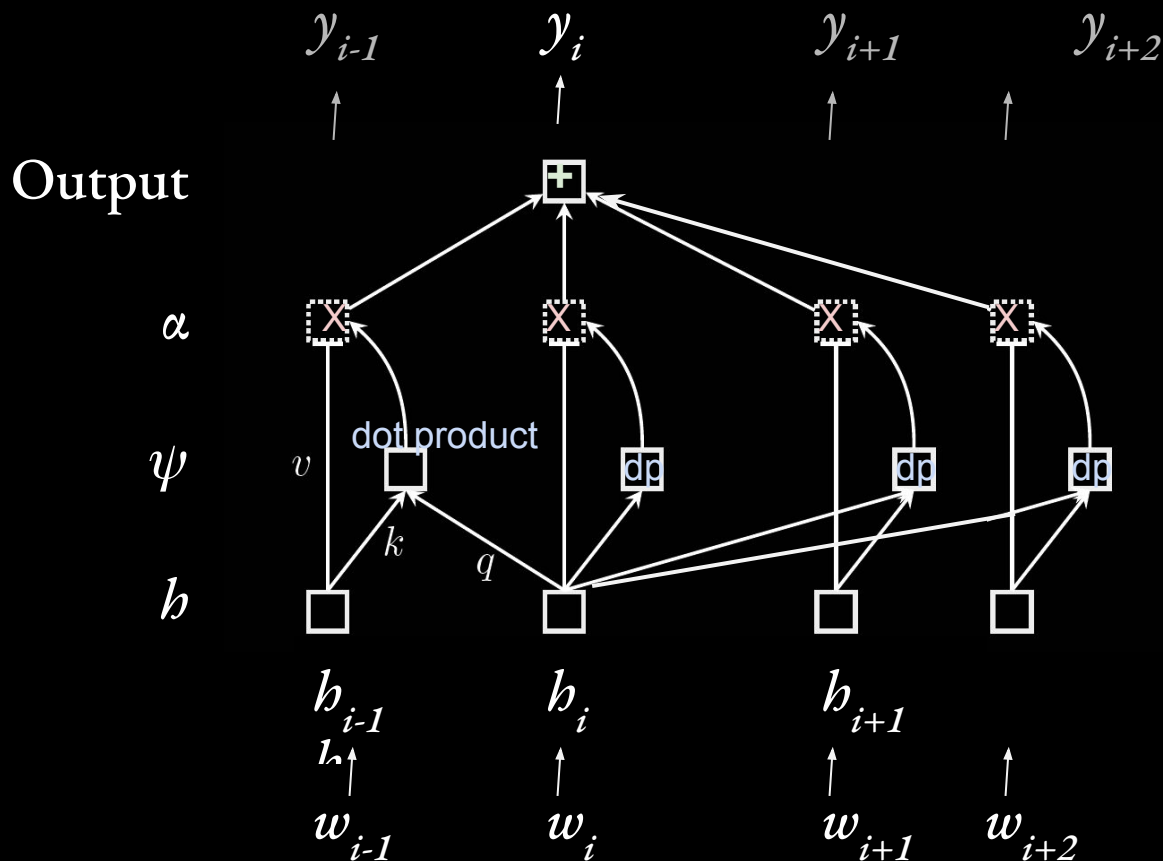
# The Transformer's Heart: Self-Attention



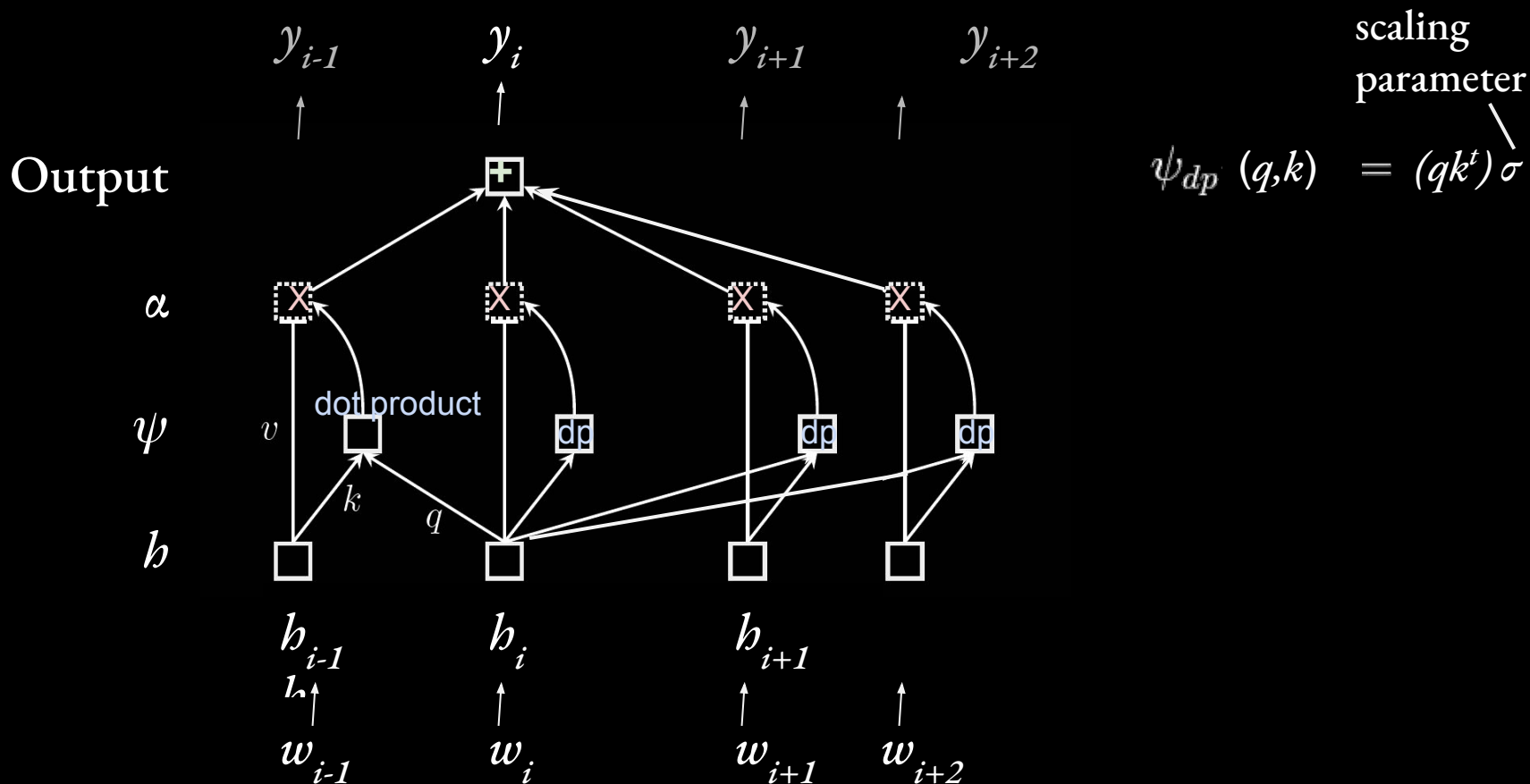
# The Transformer's Heart: Self-Attention



# The Transformer's Heart: Self-Attention



# The Transformer's Heart: Self-Attention



# Notations for Self-Attention (Matrix multiplication, Dot Product, Sequence length (s), embedding dimensions)

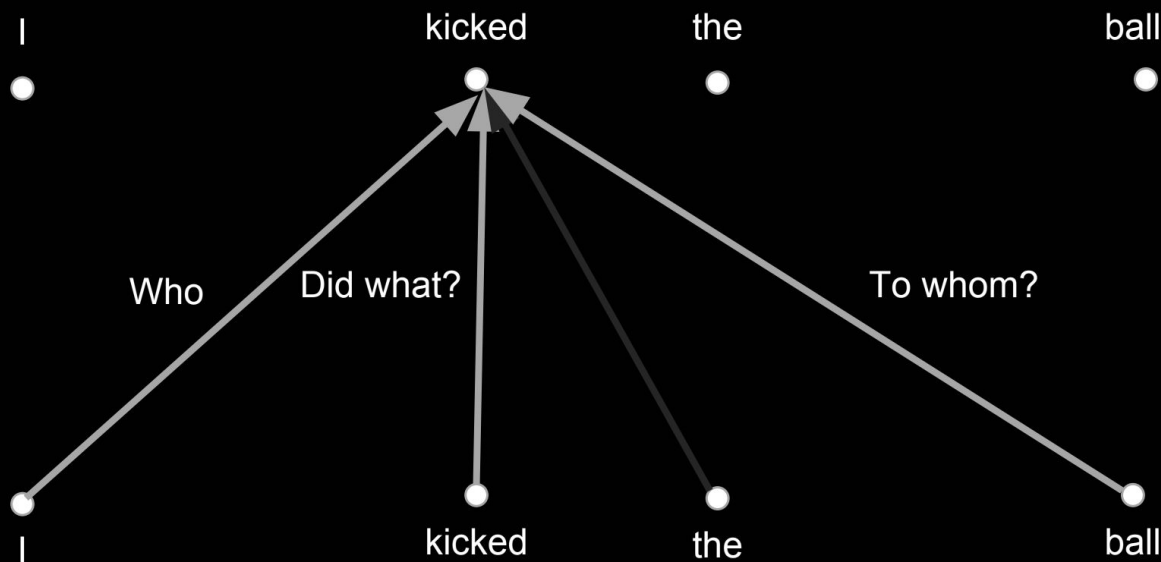
Input matrix:  $[s, d_{\text{model}}]$

# Self-Attention

$$\text{Attention}(Q, K, V) = \text{softmax}\left(\frac{QK^T}{\sqrt{d_k}}\right)V$$

# The Transformer: Beyond Self-Attention

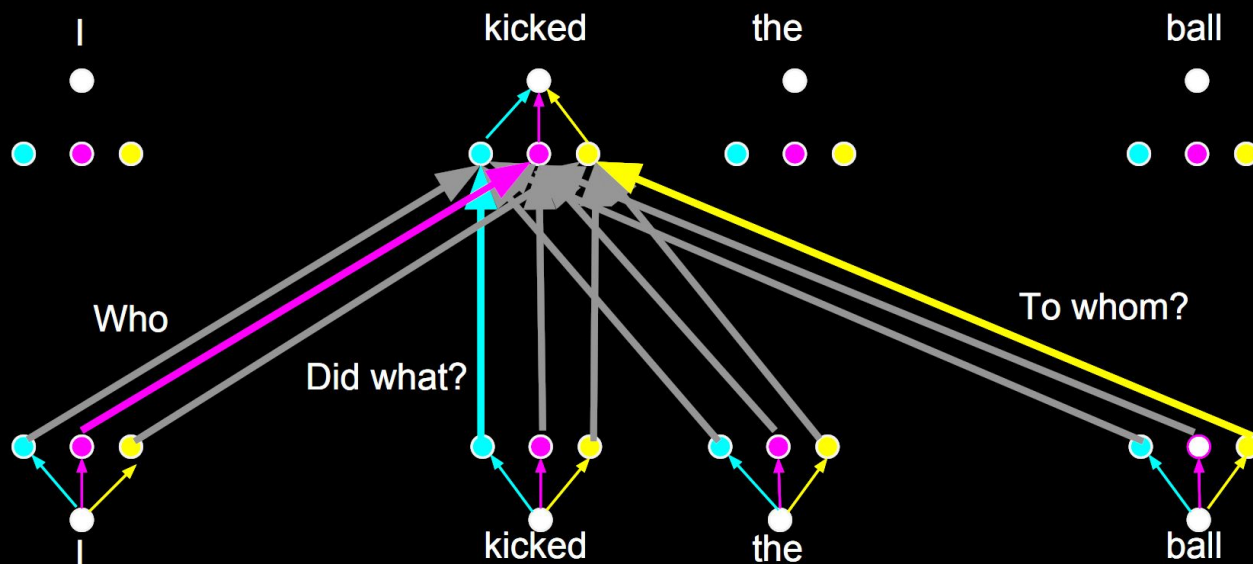
Limitation (thus far): Can't capture multiple types of dependencies between words.



# The Transformer: Beyond Self-Attention

Limitation (thus far): Can't capture multiple types of dependencies between words.

Solution: Multi-head attention





# Self-Attention: Weights

$$\text{Attention}(Q, K, V) = \text{softmax}\left(\frac{QK^T}{\sqrt{d_k}}\right)V$$

# Multi-Headed Attention

$$\text{MultiHead}(Q, K, V) = \text{Concat}(\text{head}_1, \dots, \text{head}_h)W^O$$

where  $\text{head}_i = \text{Attention}(QW_i^Q, KW_i^K, VW_i^V)$

# Multi-Headed Attention

$$\text{MultiHead}(Q, K, V) = \text{Concat}(\text{head}_1, \dots, \text{head}_h)W^O$$

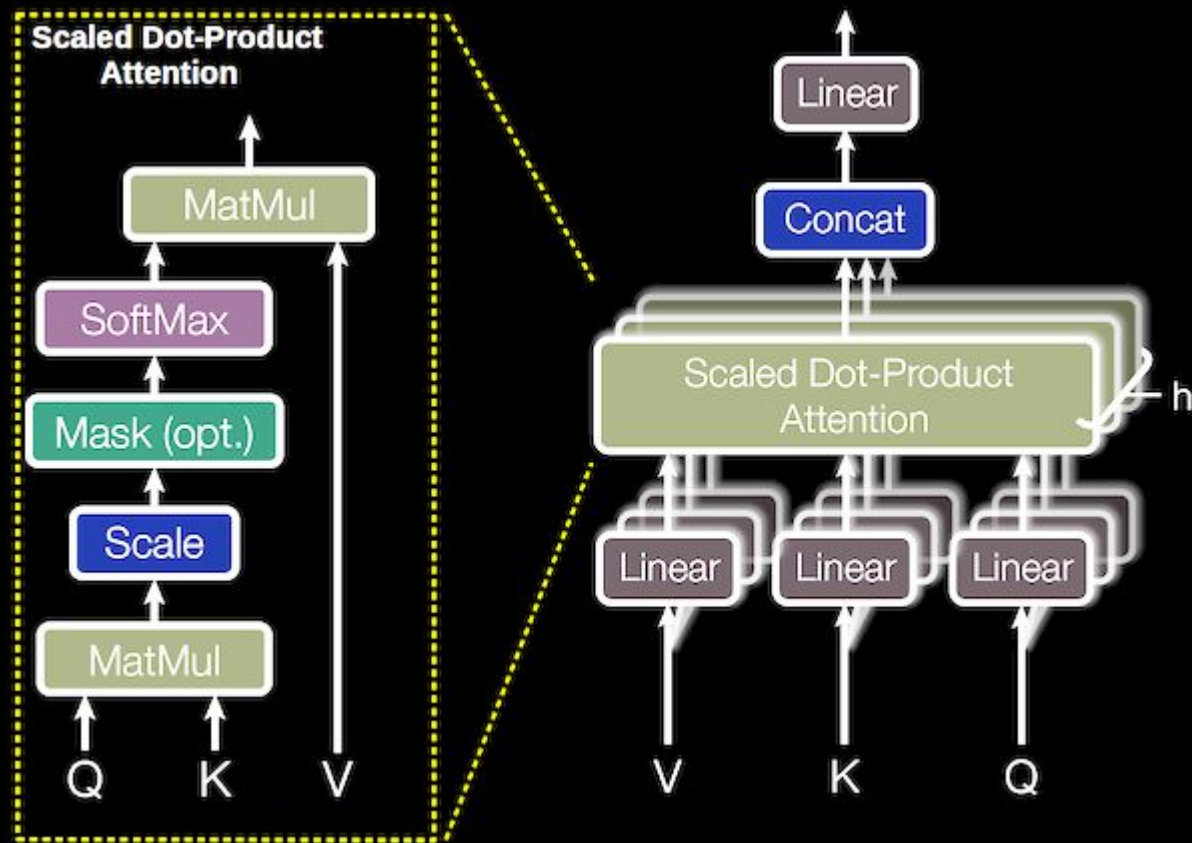
where  $\text{head}_i = \text{Attention}(QW_i^Q, KW_i^K, VW_i^V)$

Linear layer:

$$W^T X$$

One set of weights for each of K, Q, and V

# The Transformer: Multi-headed Attention



# Self-Attention in PyTorch

```
import nn.functional as f

class SelfAttention(nn.Module):
    def __init__(self, h_dim:int):
        self.Q = nn.Linear(h_dim, h_dim) #1 head
        self.K = nn.Linear(h_dim, h_dim)
        self.V = nn.Linear(h_dim, h_dim)

    def forward(hidden_states:torch.Tensor):
        v = self.V(hidden_states)
        k = self.K(hidden_states)
        q = self.Q(hidden_states)
        attn_scores = torch.matmul(q, k.T)
        attn_probs = f.Softmax(attn_scores)

        context = torch.matmul(attn_probs, v)
        return context
```

$$\psi_{dp}(q,k) = (qk^t)\sigma$$

Linear layer:

$$W^T X$$

One set of weights  
for each of K, Q,  
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# Self-Attention in PyTorch

```
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        self.Q = nn.Linear(h_dim, h_dim) #1 head
        self.K = nn.Linear(h_dim, h_dim)
        self.V = nn.Linear(h_dim, h_dim)
        self.dropout = nn.dropout(p=0.1)

    def forward(hidden_states:torch.Tensor):
        v = self.V(hidden_states)
        k = self.K(hidden_states)
        q = self.Q(hidden_states)
        attn_scores = torch.matmul(q, k.T)
        attn_probs = f.Softmax(attn_scores)
        attn_probs = self.dropout(attn_probs)
        context = torch.matmul(attn_probs, v)
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```

$$\psi_{dp}(q,k) = (qk^t)\sigma$$

Linear layer:

$$W^T X$$

One set of weights  
for each of K, Q,  
and V

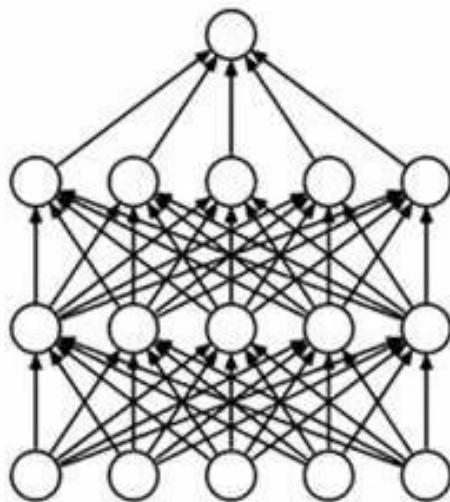
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```
import nn.functional as f
class SelfAttention(nn.Module):
```

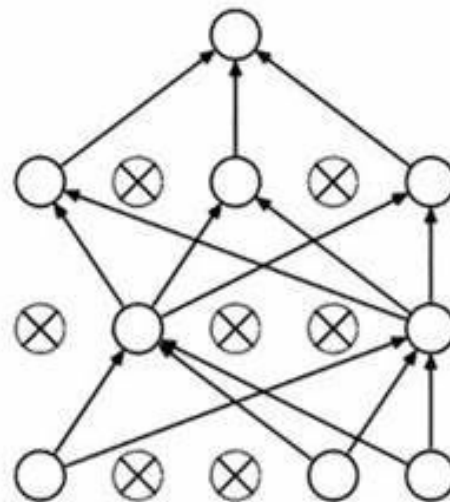
```
    def __init__(
        self.Q =
        self.K =
        self.V =
        self.drop
```

```
    def forward(h
        v = self.
        k = self.
        q = self.
        attn_scor
        attn_prob
```

```
        attn_probs = self.dropout(attn_probs)
        context = torch.matmul(attn_probs, v)
        return context
```



(a) Standard Neural Net



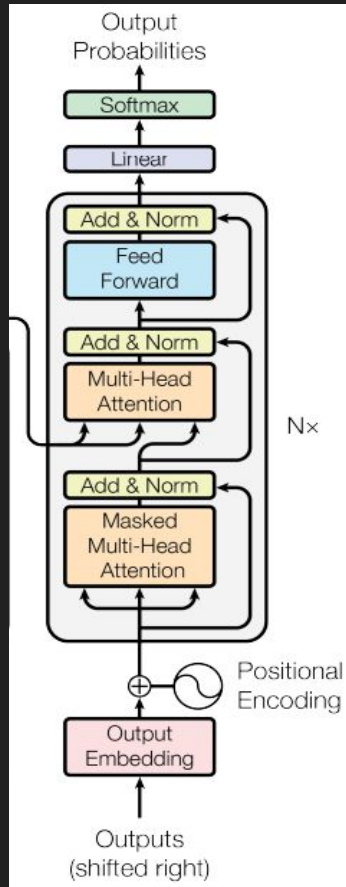
(b) After applying dropout.

$$= (qk^t) \sigma$$

ayer:

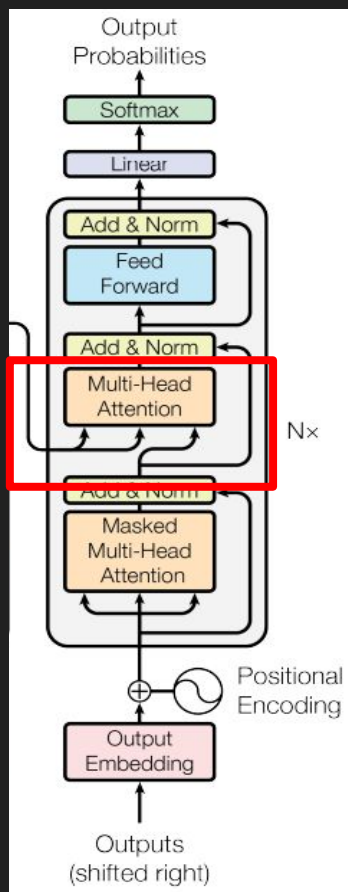
of weights  
of for K,

# Decoder

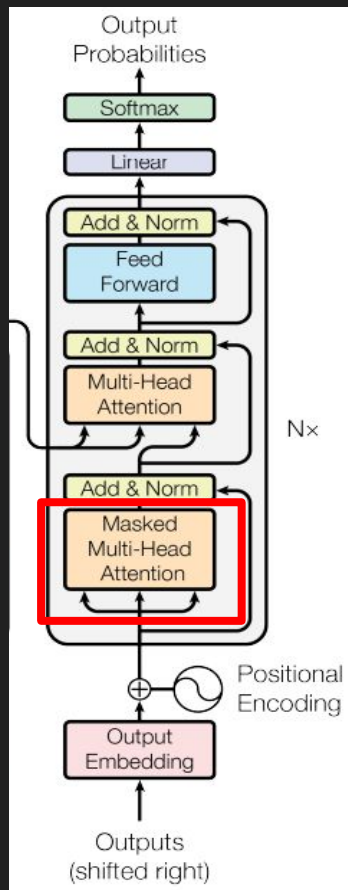




# Decoder: Cross Attention



# Decoder: Masked Multi-Head Attention

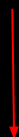


# Masked Multi-Head Attention

# Training

[English]

I love hiking.



[Italian]

Adoro le escursioni.

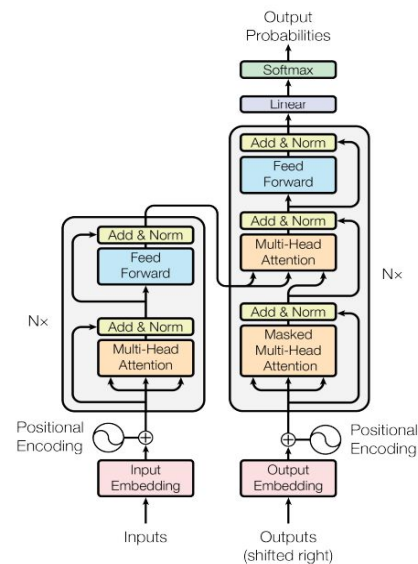


Figure 1: The Transformer - model architecture.

# Training

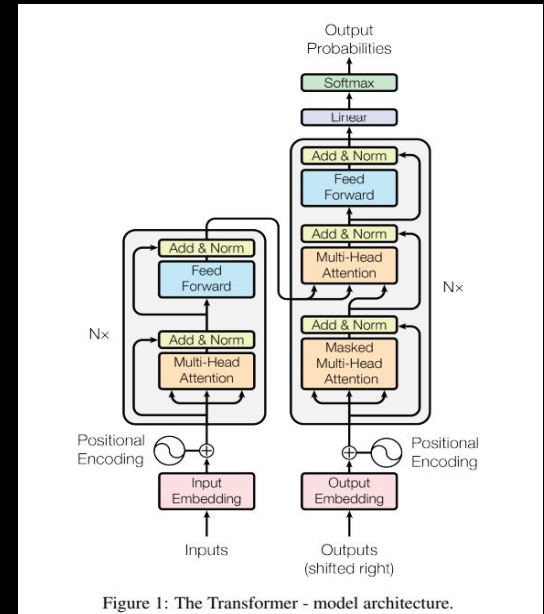
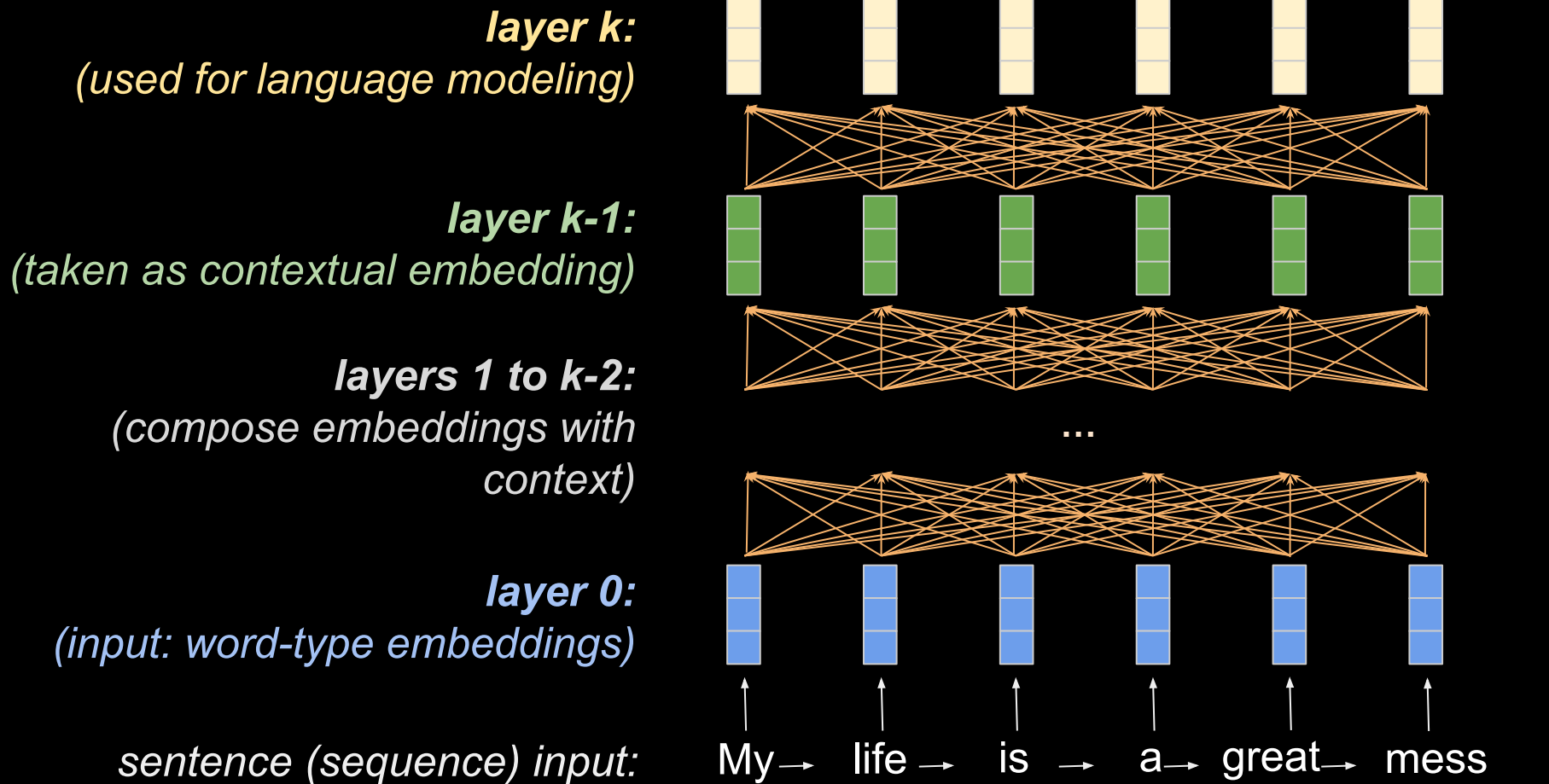


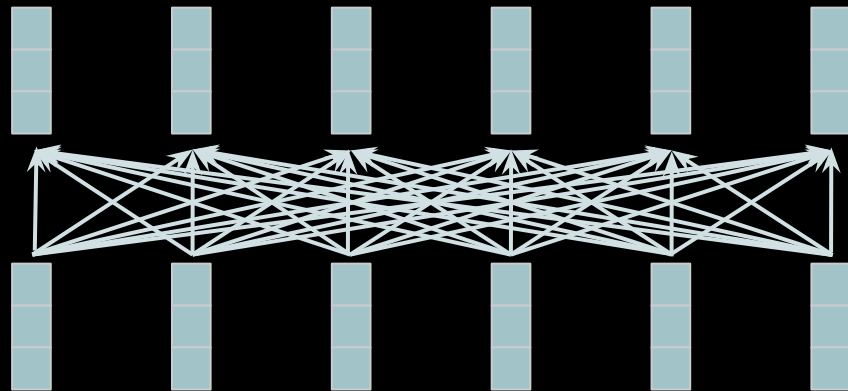
Figure 1: The Transformer - model architecture.

# Transformer Language Models: Uses multiple layers of a **transformer**



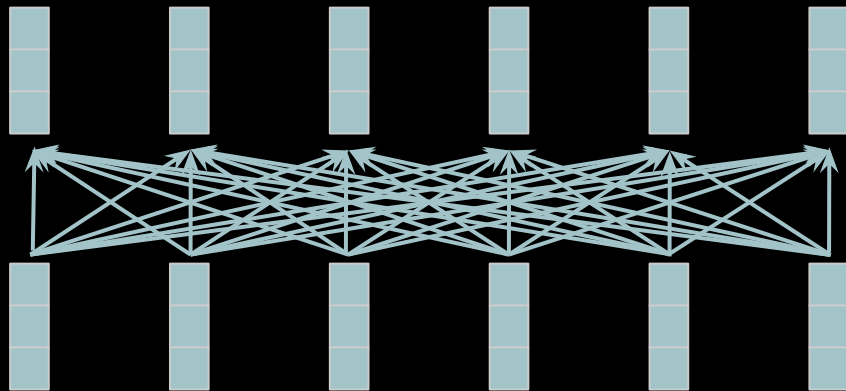
## **Auto-encoder (MLM):**

- Connections go both directions.
- Task is predict word in middle:  
 $p(w_i \mid \dots, p_{w_i-2}, w_{i-1}, w_{i+1}, w_{i+2} \dots)$
- Better for:
  - embeddings
  - fine-tuning (transfer learning)



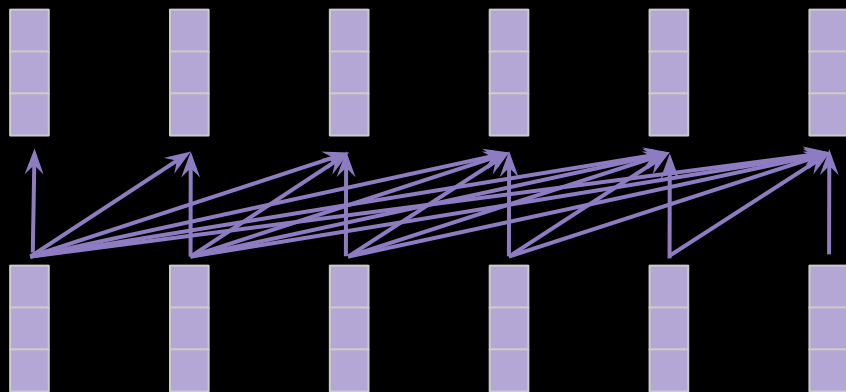
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- Better for:
  - embeddings
  - fine-tuning (transfer learning)



## **Auto-regressor (generator):**

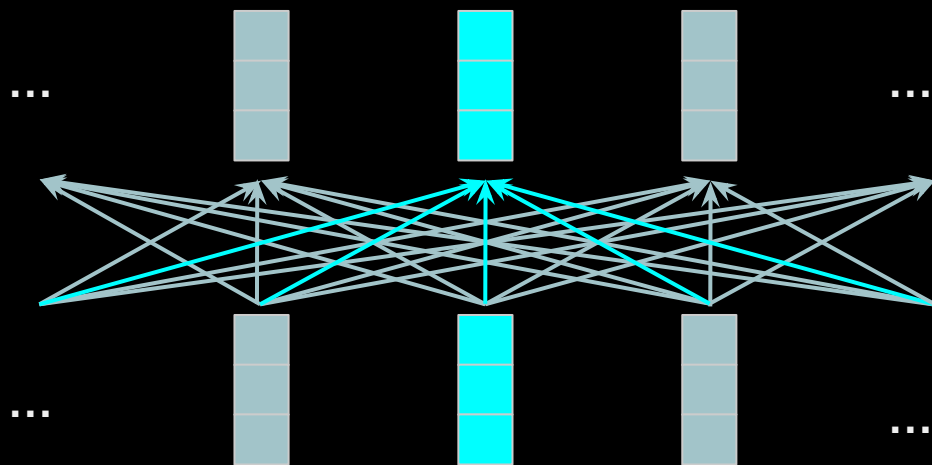
- Connections go forward only
- Task is predict word next word:  
 $p(w_i | w_{i-1}, w_{i-2}, \dots)$
- Better for:
  - generating text
  - zero-shot learning





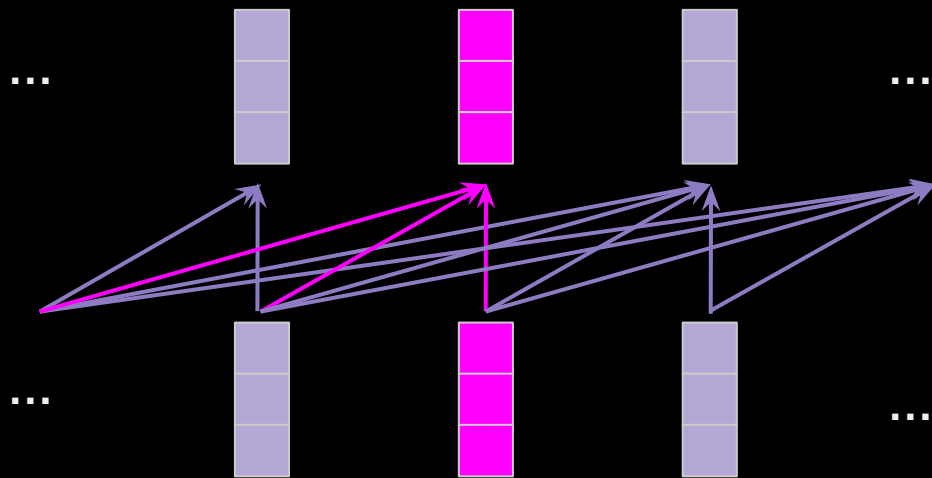
## **Auto-encoder (MLM):**

- Connections go both directions.
- Task is predict word in middle:  
 $p(w_i | \dots, p_{w_i-2}, w_{i-1}, w_{i+1}, w_{i+2} \dots)$
- Better for:
  - embeddings
  - fine-tuning (transfer learning)

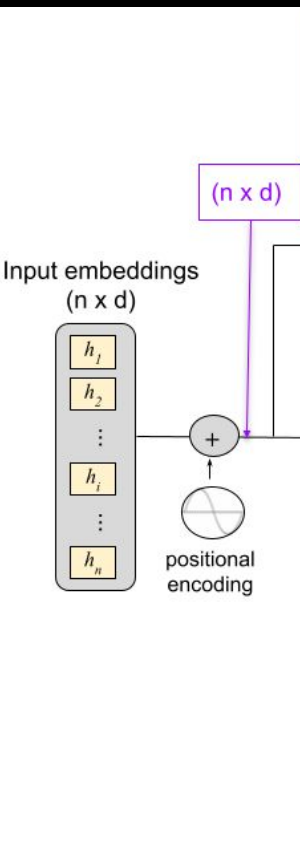


## **Auto-regressor (generator):**

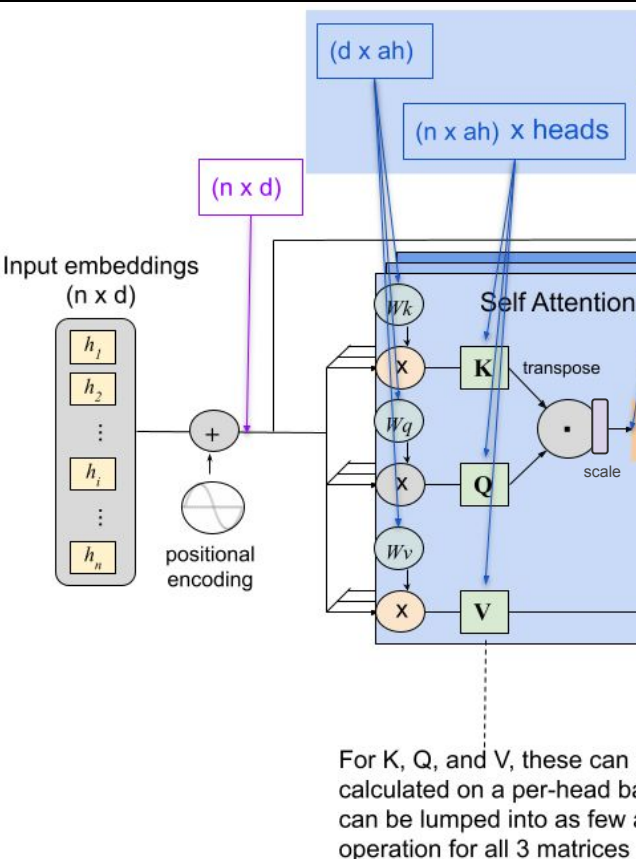
- Connections go forward only
- Task is predict word next word:  
 $p(w_i | w_{i-1}, w_{i-2}, \dots)$
- Better for:
  - generating text
  - zero-shot learning



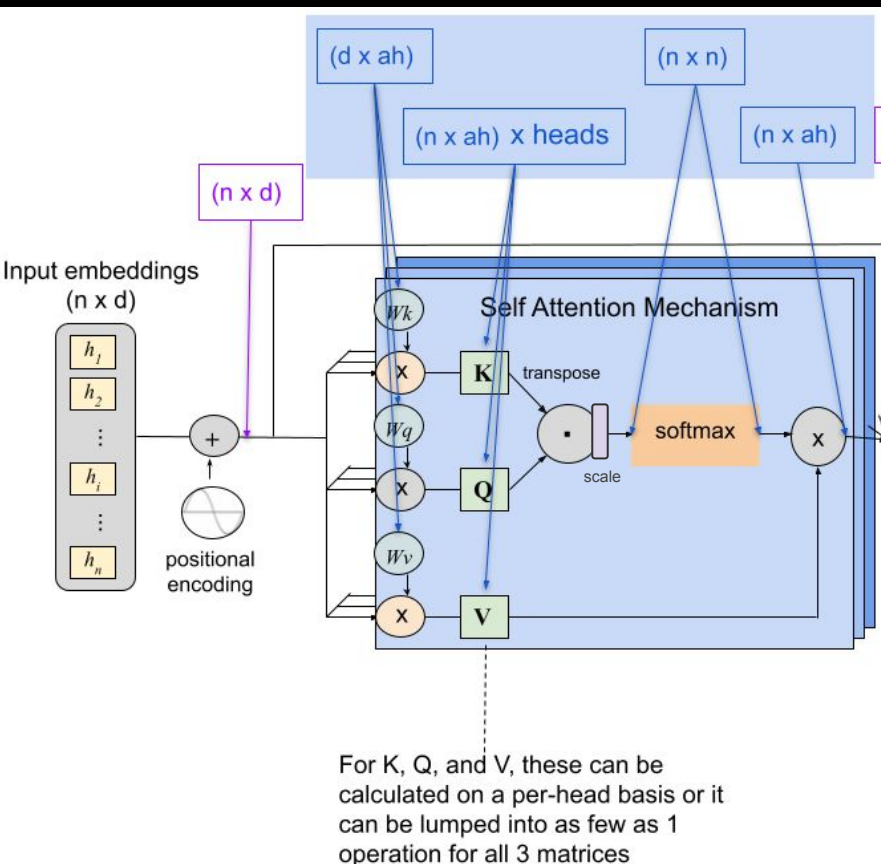
# Detailed Overview of (HuggingFace) Transformer Matrices and Computation



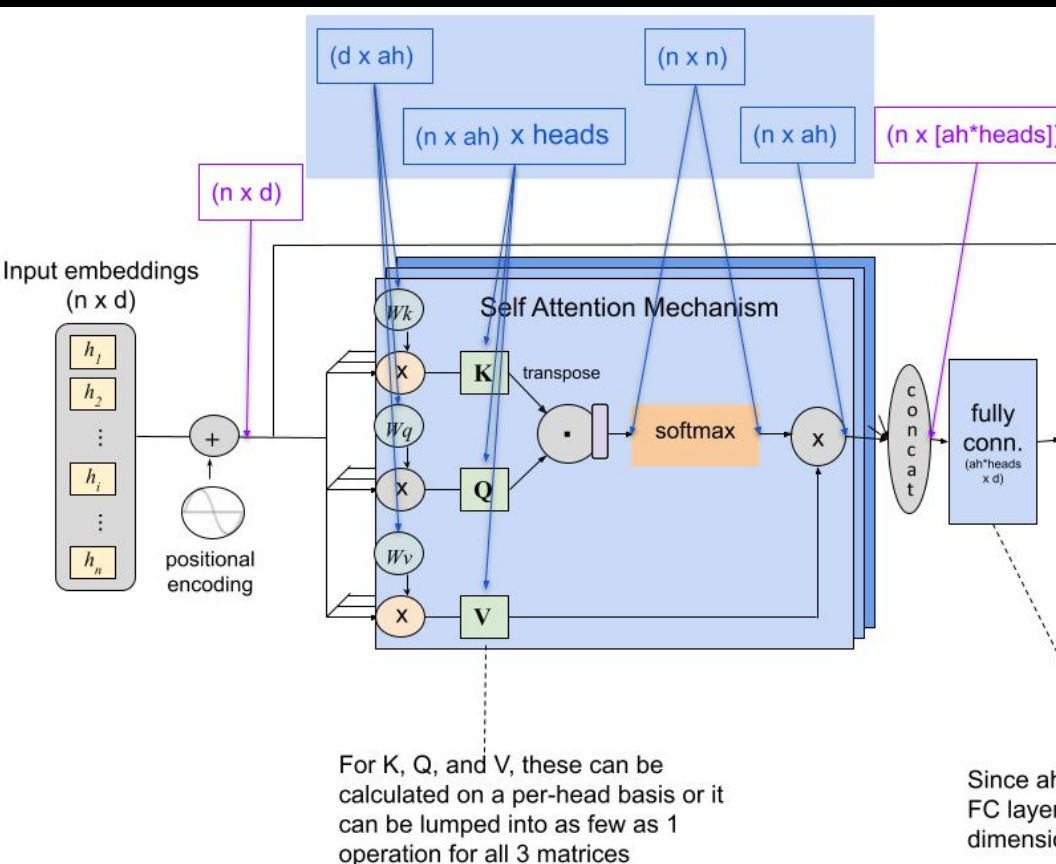
# Detailed Overview of (HuggingFace) Transformer Matrices and Computation



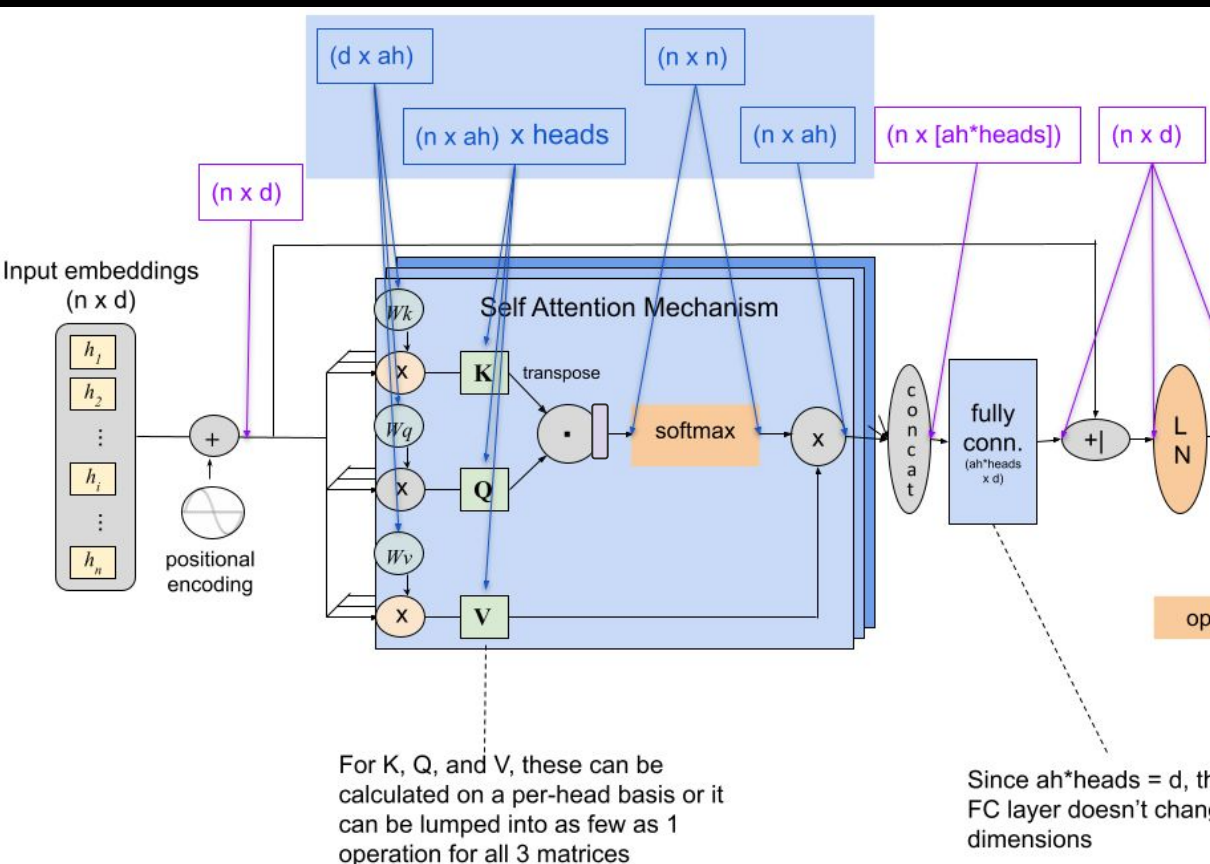
# Detailed Overview of (HuggingFace) Transformer Matrices and Computation



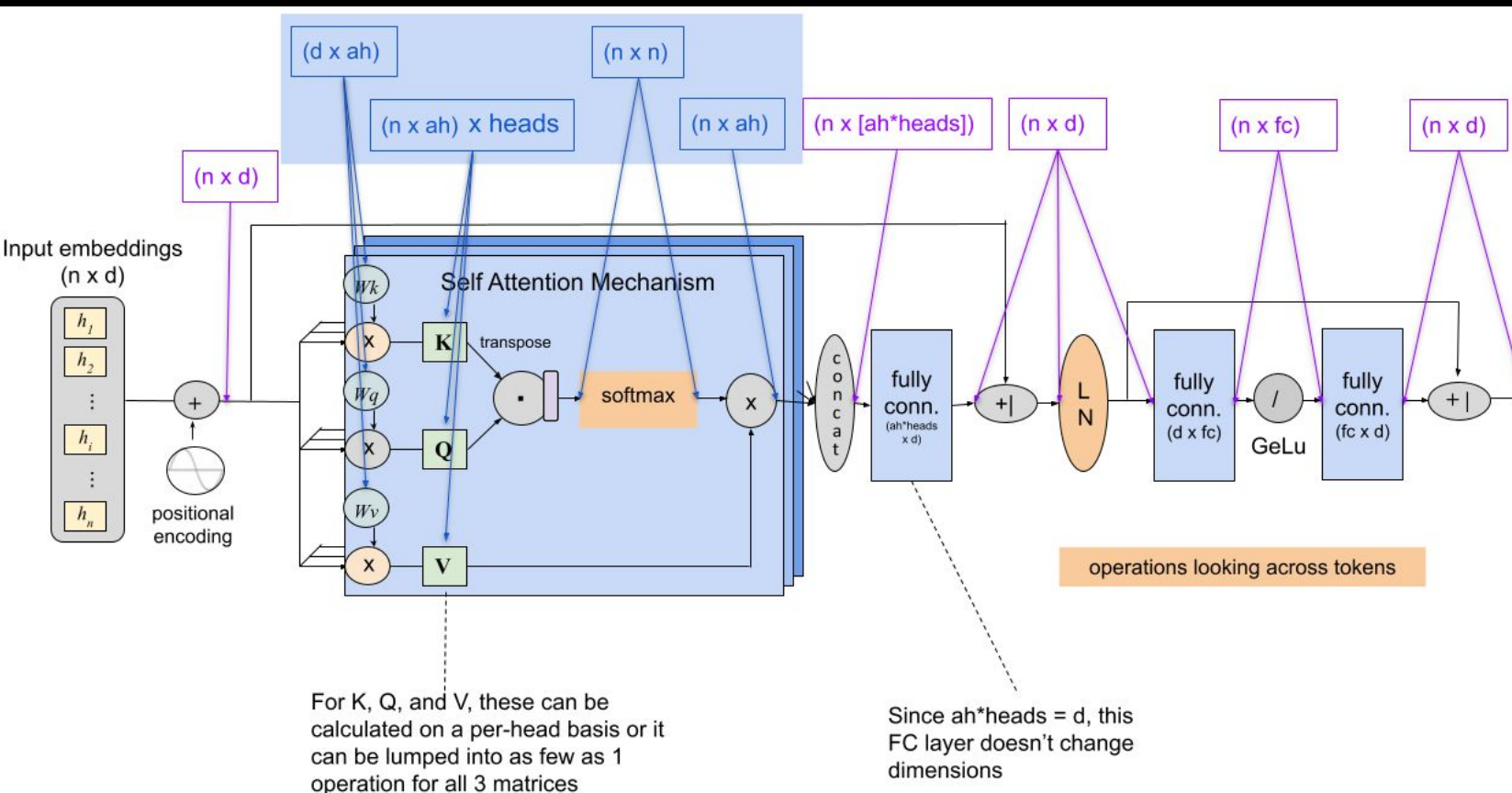
# Detailed Overview of (HuggingFace) Transformer Matrices and Computation



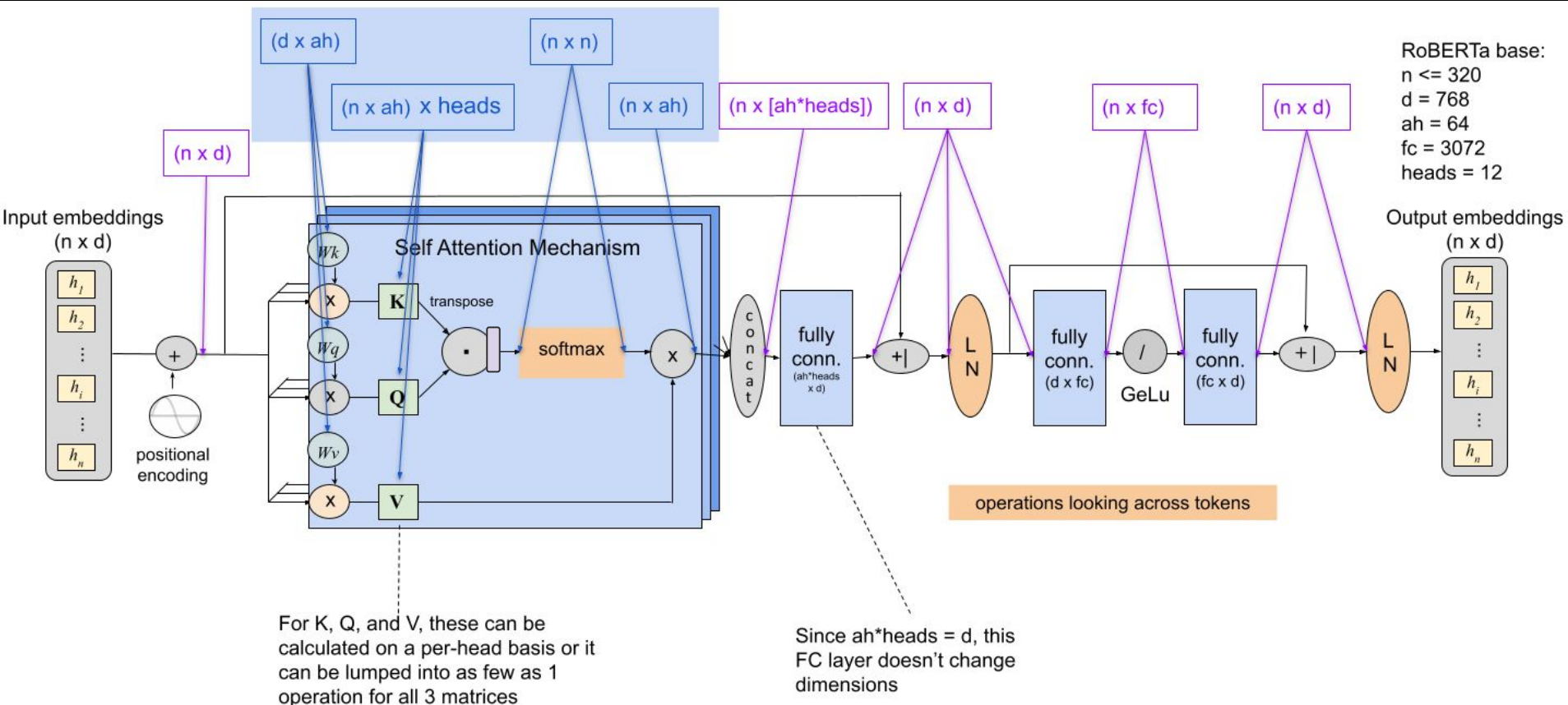
# Detailed Overview of (HuggingFace) Transformer Matrices and Computation



# Detailed Overview of (HuggingFace) Transformer Matrices and Computation



# Detailed Overview of (HuggingFace) Transformer Matrices and Computation





# Hugging Face or AllenNLP

<https://github.com/huggingface/transformers>

```
#example for getting embeddings
from transformers import BertModel, PreTrainedTokenizerFast, pipeline

bert_tokenizer = PreTrainedTokenizerFast.from_pretrained('google-bert/bert-base-uncased')
bert_model = BertModel.from_pretrained('google-bert/bert-base-uncased')
pipe = pipeline('feature-extraction', model=bert_model, tokenizer=bert_tokenizer)
emb = pipe(text)
print(emb[0][0])
```

[https://docs.allennlp.org/v2.10.1/api/modules/transformer/transformer\\_module/](https://docs.allennlp.org/v2.10.1/api/modules/transformer/transformer_module/)

# Transformer (as of 2017)

“WMT-2014” Data Set. BLEU scores:

|              | EN-DE       | EN-FR       |
|--------------|-------------|-------------|
| GNMT (orig)  | 24.6        | 39.9        |
| ConvSeq2Seq  | 25.2        | 40.5        |
| Transformer* | <b>28.4</b> | <b>41.8</b> |

# Transformers as of 2023

General Language Understanding Evaluations:

<https://gluebenchmark.com/leaderboard>

<https://super.gluebenchmark.com/leaderboard/>

## ChatGPT



ChatGPT is an artificial intelligence chatbot developed by OpenAI and launched in November 2022. It is built on top of OpenAI's GPT-3.5 and GPT-4 families of large language models and has been fine-tu...



# Transformers as of 2023

Machine  
Translation

Web  
Search

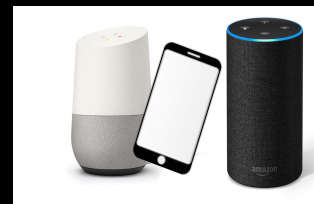
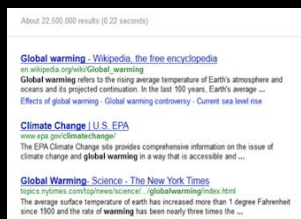
Sentiment  
Analysis

Document  
Classification

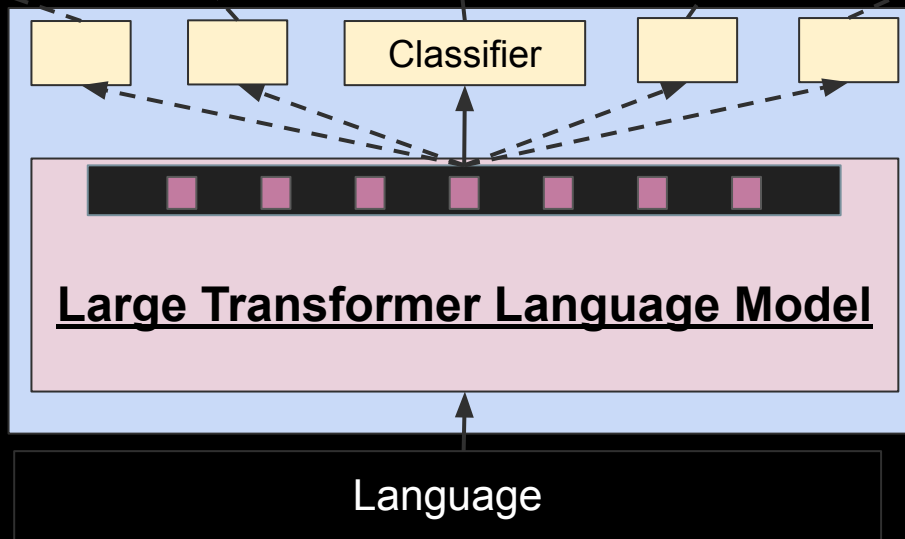
Assistant,  
QA

...

absolutamente  
me gustaría ir  
de excursión



Soni, N., Matero, M.,  
Balasubramanian, N., &  
Schwartz, H. (2022, May).  
Human Language Modeling. In  
*Findings of the Association for  
Computational Linguistics: ACL  
2022* (pp. 622-636).

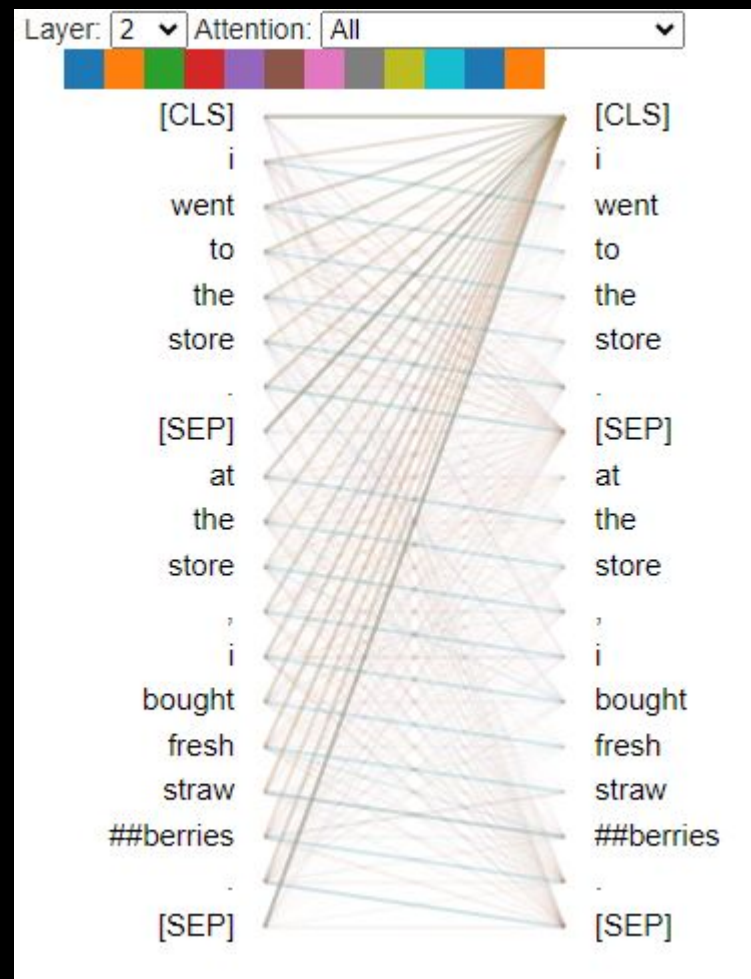


(NLP System)



# Bert: Attention by Layers

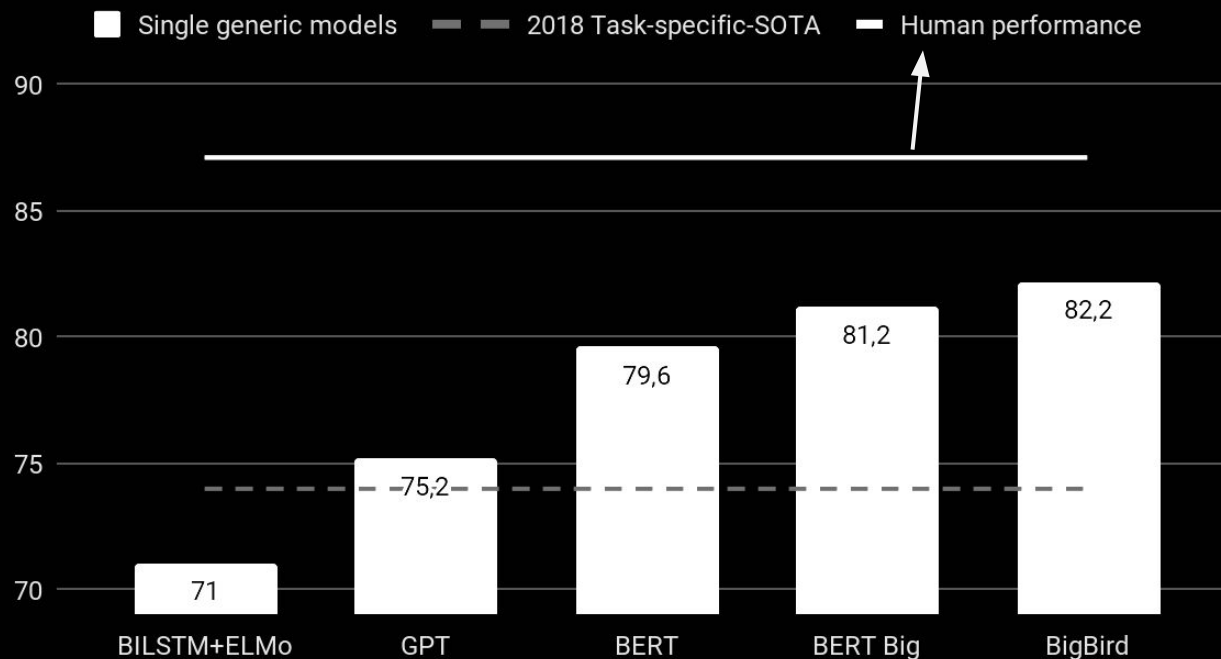
<https://colab.research.google.com/drive/1vIOJ1lhdujVjfH857hvYKIdKPTD9Kid8>



(Vig, 2019)

# BERT Performance: e.g. Question Answering

GLUE scores evolution over 2018-2019



<https://rajpurkar.github.io/SQuAD-explorer/>

# The Transformer: Take Away

## Challenges to sequential representation learning

- Capture long-distance dependencies  
*Self-attention treats far away words similar to those close.*
- Preserving sequential distances / periodicity  
*Positional embeddings encode distances/periods.*
- Capture multiple relationships  
*Multi-headed attention enables multiple compositions.*
- Easy to parallelize -- don't need sequential processing.  
*Entire layer can be computed at once. Is only matrix multiplications + standardizing.*

# Part 3: Applying Transformer LMs

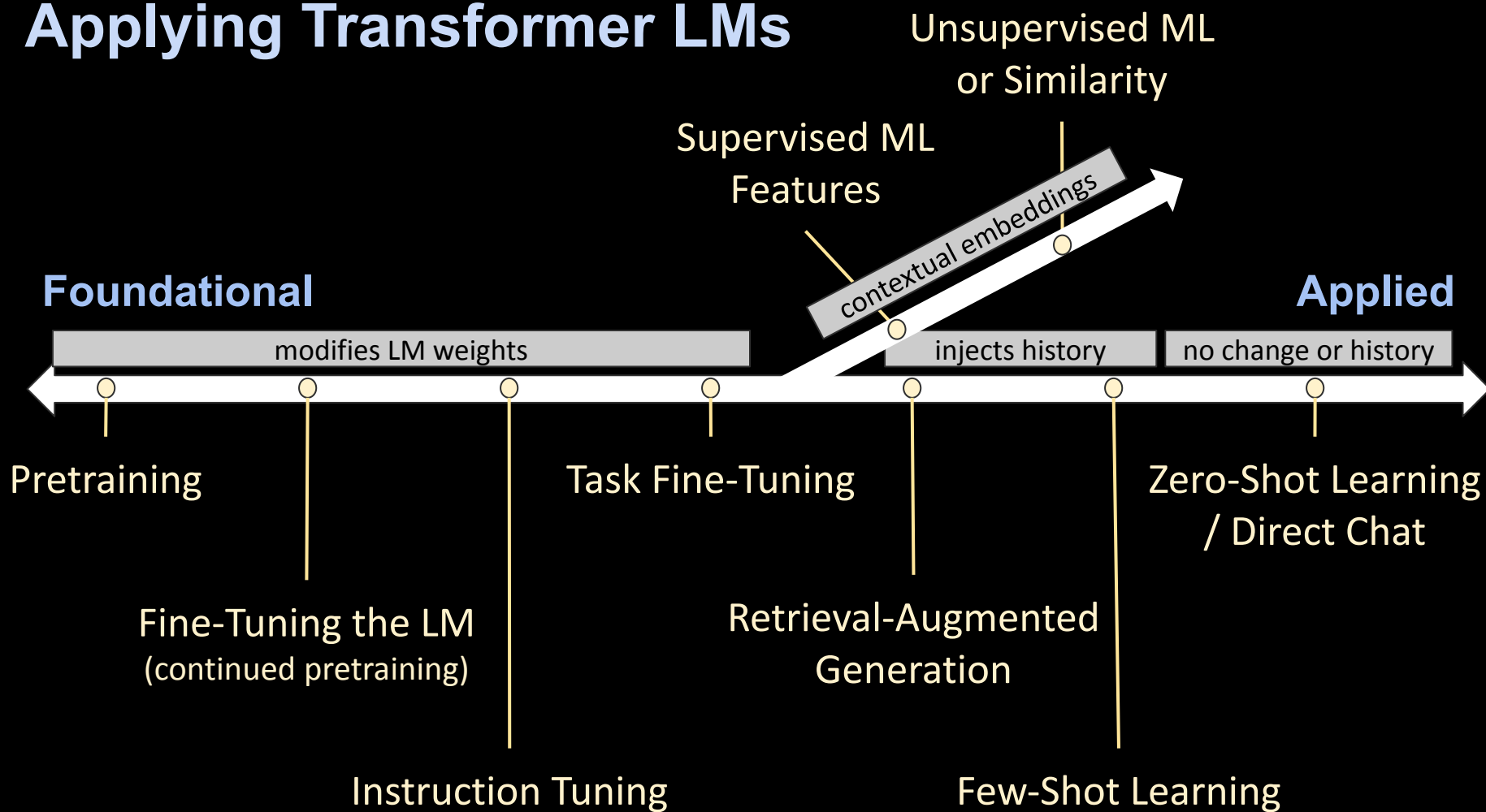
**Foundational**

**Applied**

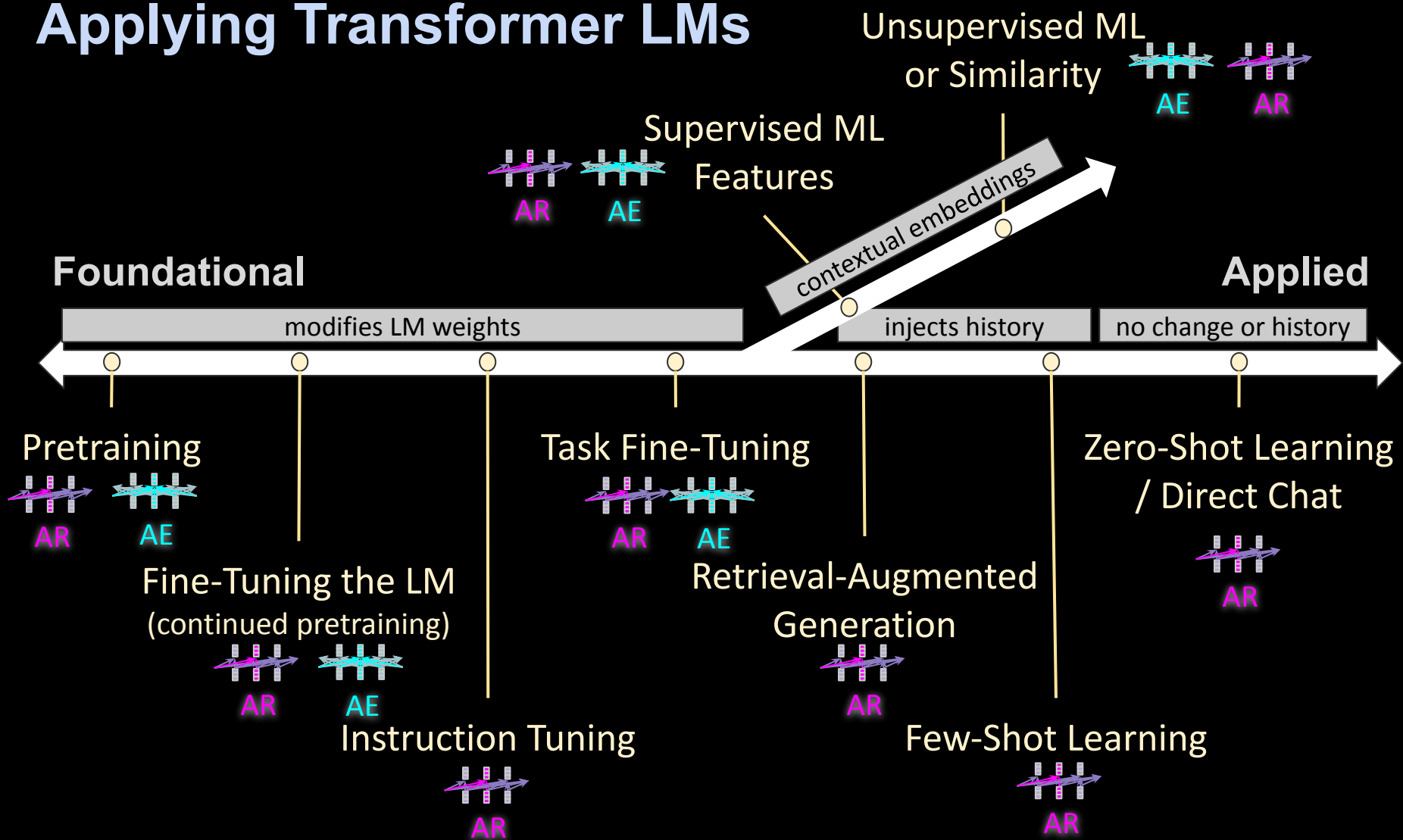




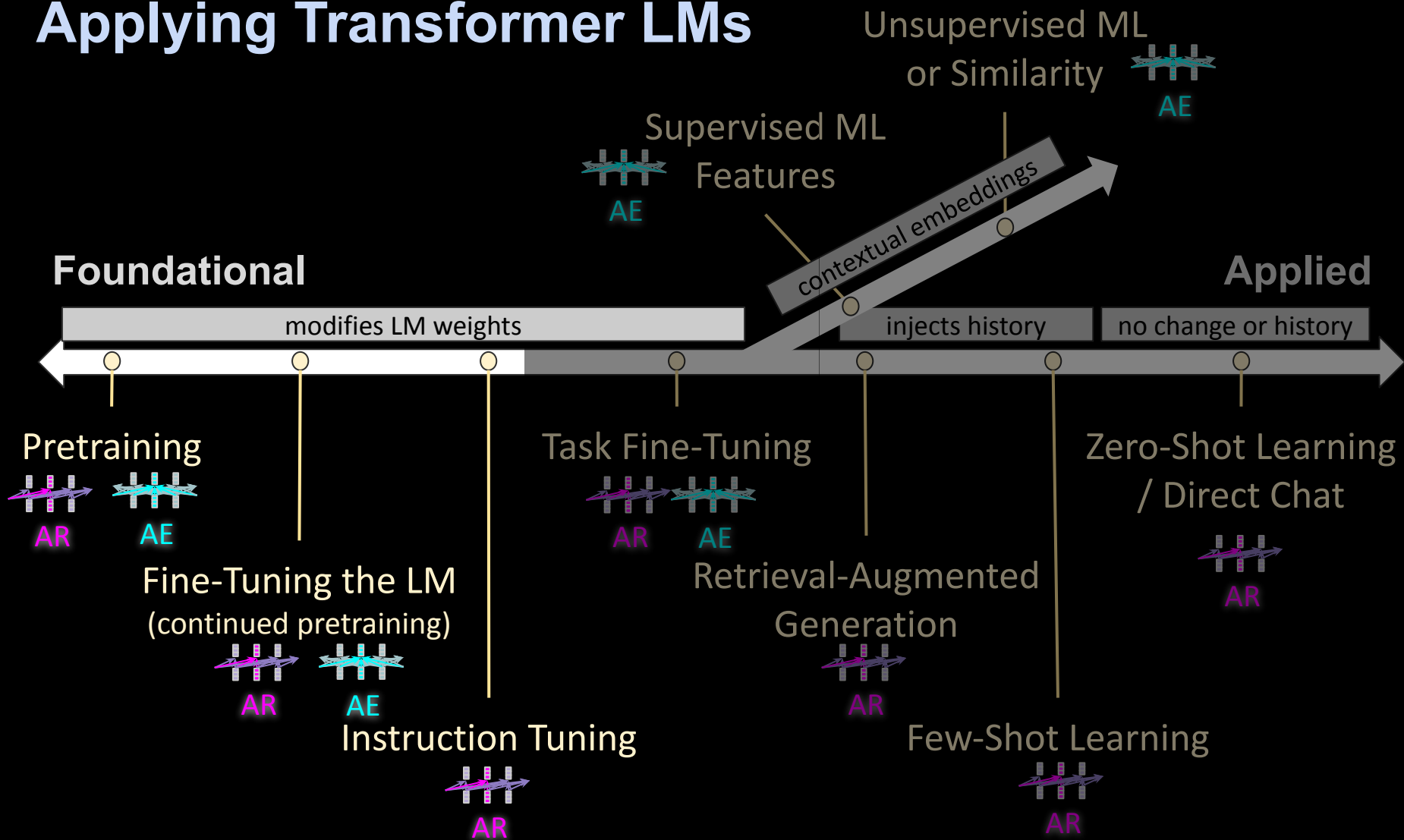
# Applying Transformer LMs



# Applying Transformer LMs



# Applying Transformer LMs



# Pretraining; FTing the LM; Instruction Tuning

***softmax for LM:***

***layer  $k$ :***

*(used for language modeling)*

***layer  $k-1$ :***

*(taken as contextual embedding)*

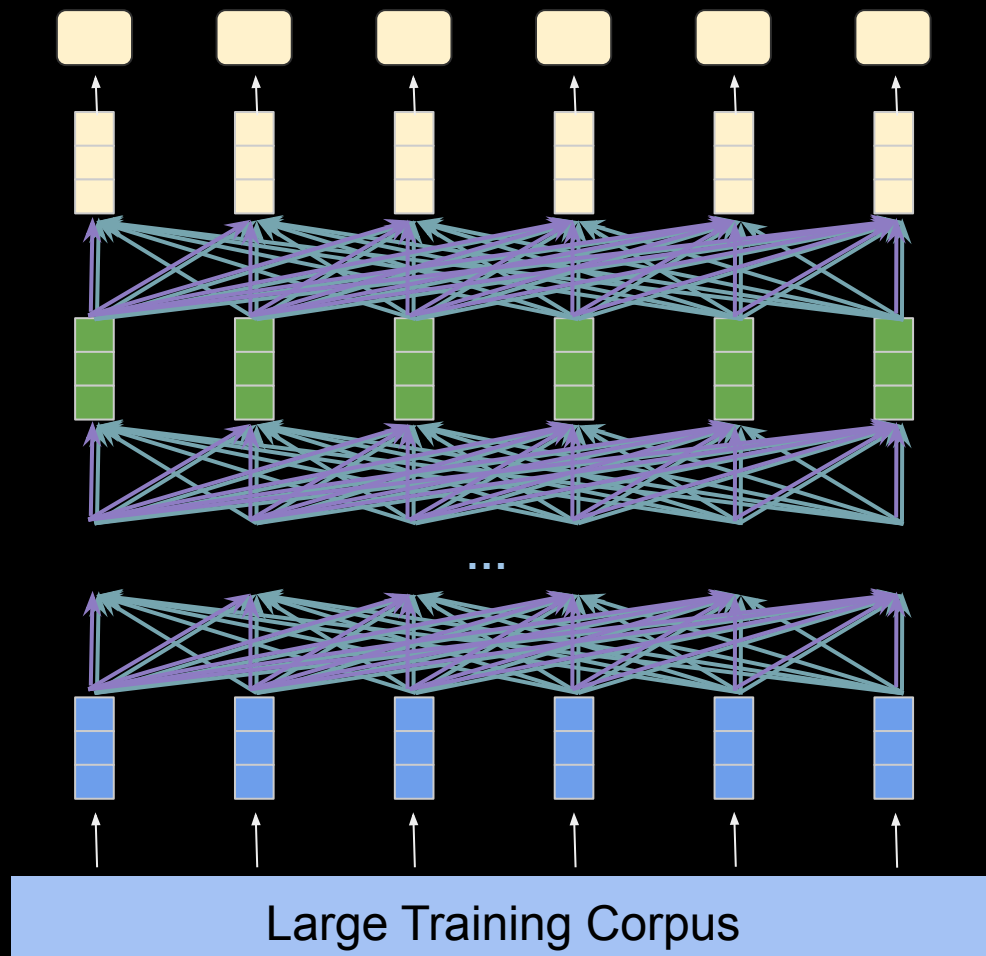
***layers 1 to  $k-2$ :***

*(compose embeddings with context)*

***layer 0:***

*(input: word-type embeddings)*

*sentence (sequence) input:*



# Pretraining; FTing the LM; Instruction Tuning

**softmax for LM:**

**layer  $k$ :**

*(used for language modeling)*

**layer  $k-1$ :**

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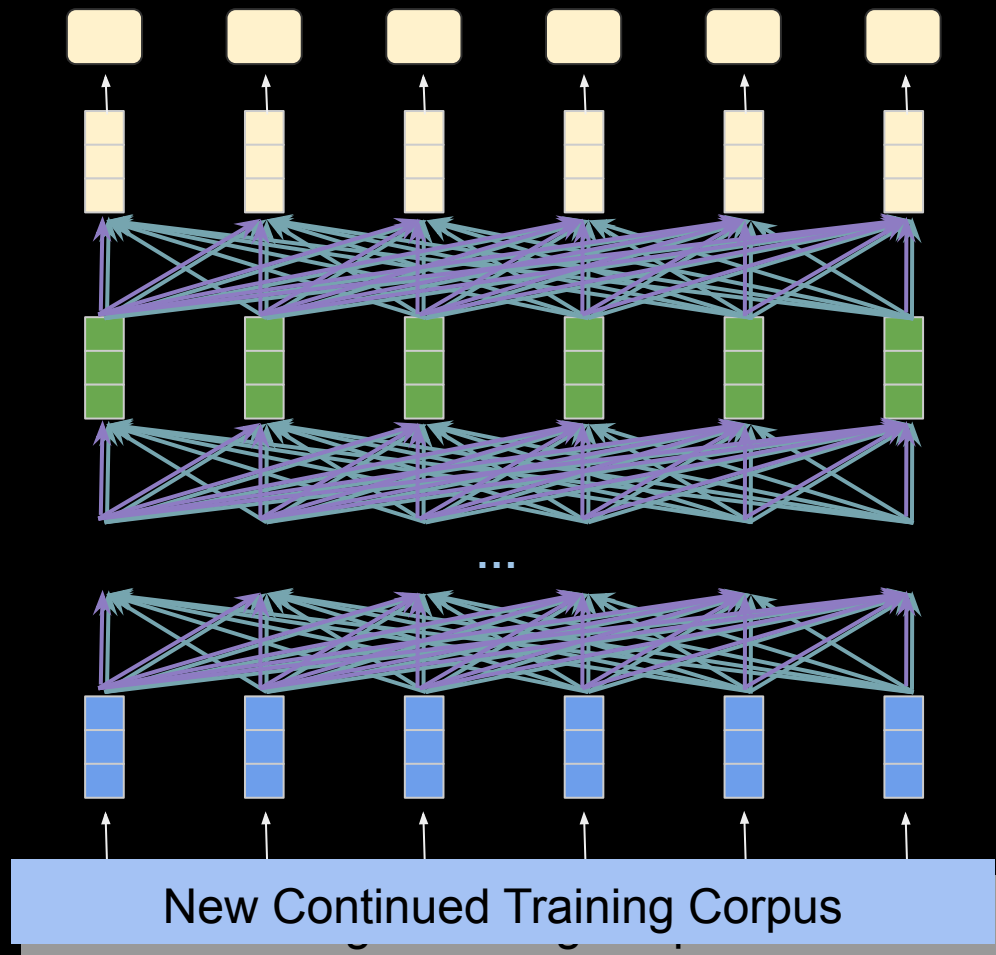
**layers 1 to  $k-2$ :**

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# Pretraining; FTing the LM; Instruction Tuning

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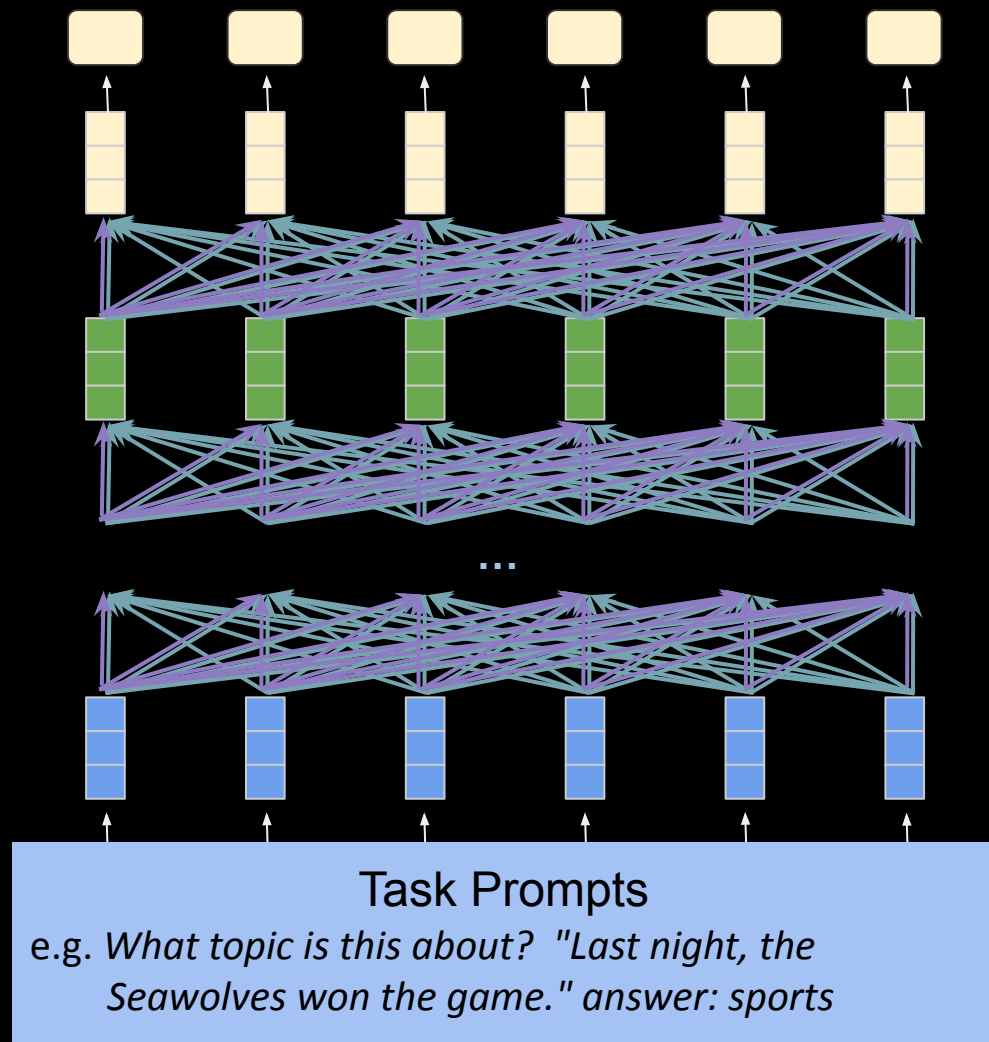
**layers 1 to  $k-2$ :**

*(compose embeddings with context)*

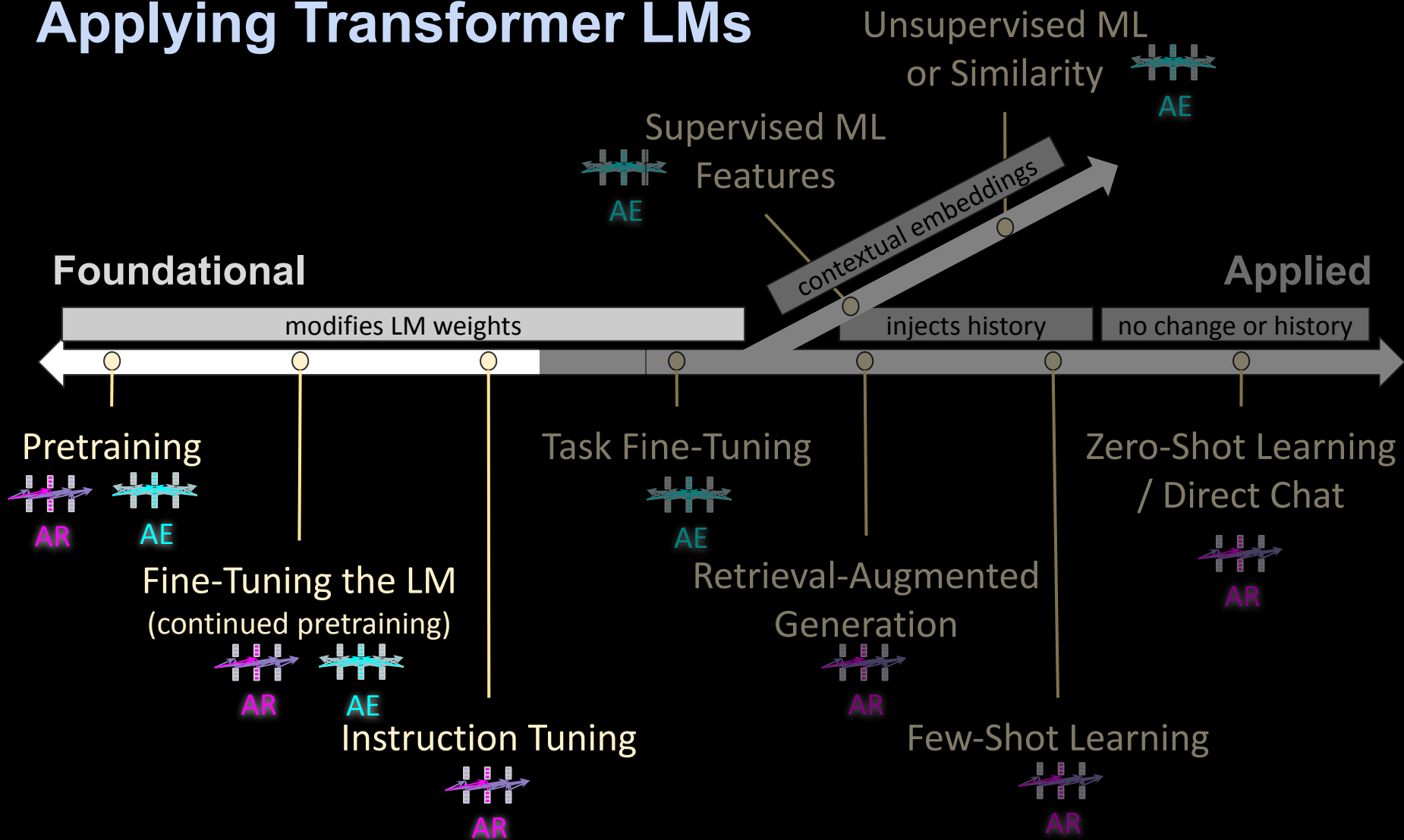
**layer 0:**

*(input: word-type embeddings)*

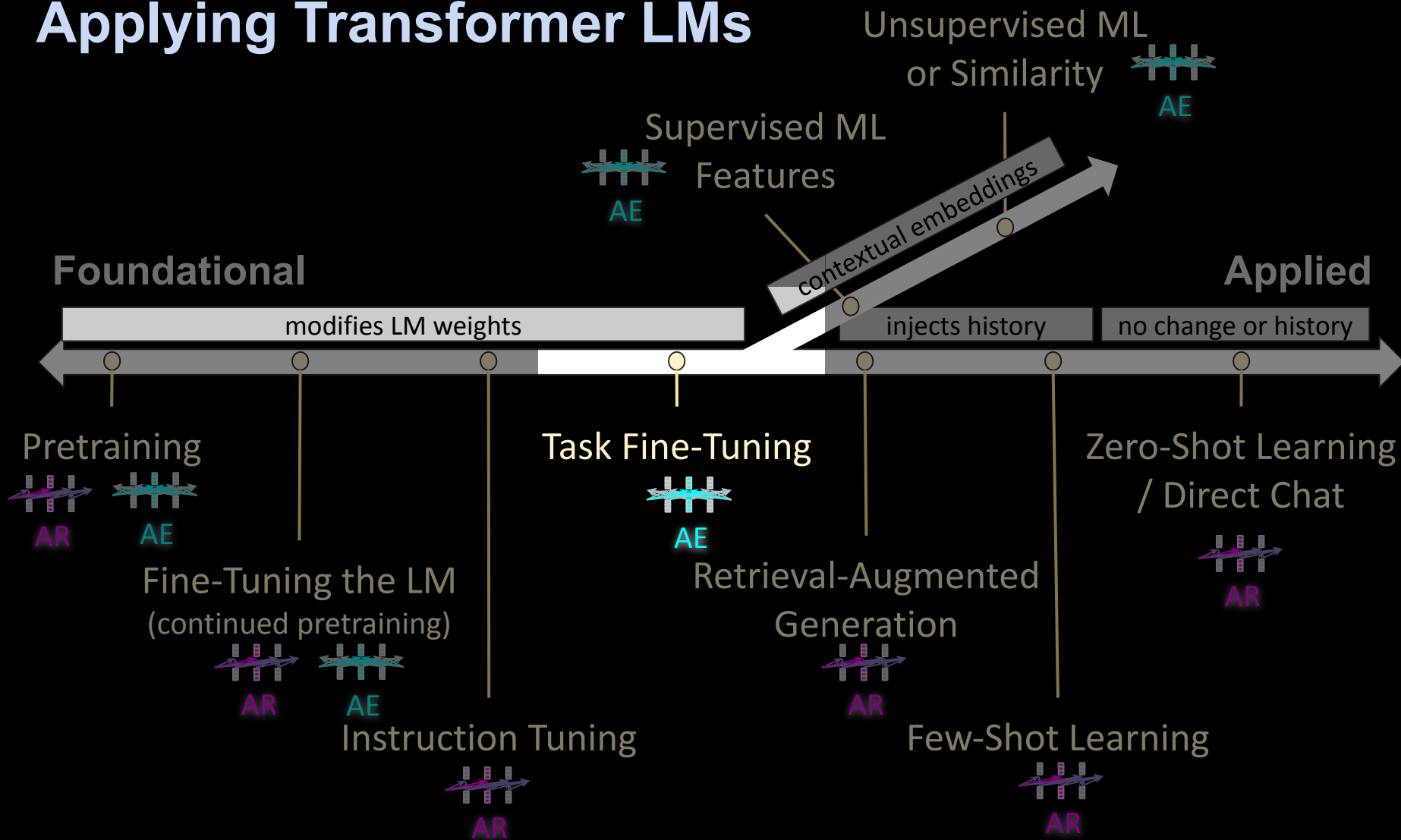
*sentence (sequence) input:*



# Applying Transformer LMs



# Applying Transformer LMs





# Task Fine-Tuning

~~softmax for LM:~~

~~layer  $k$~~

~~(used for language modeling)~~

**layer  $k-1$ :**

(taken as contextual embedding)

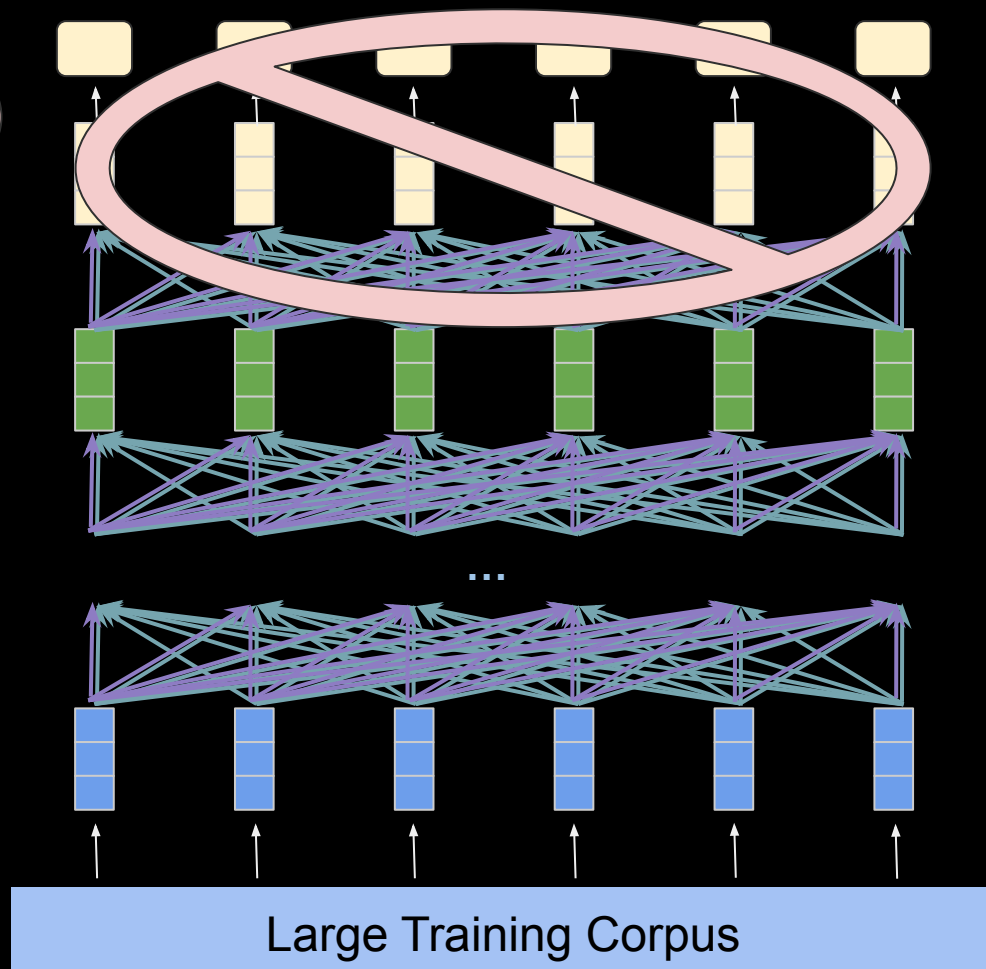
**layers 1 to  $k-2$ :**

(compose embeddings with context)

**layer 0:**

(input: word-type embeddings)

sentence (sequence) input:



# Task Fine-Tuning

***classifier or regressor:***  
*(e.g. sentiment, topic classification, etc.)*

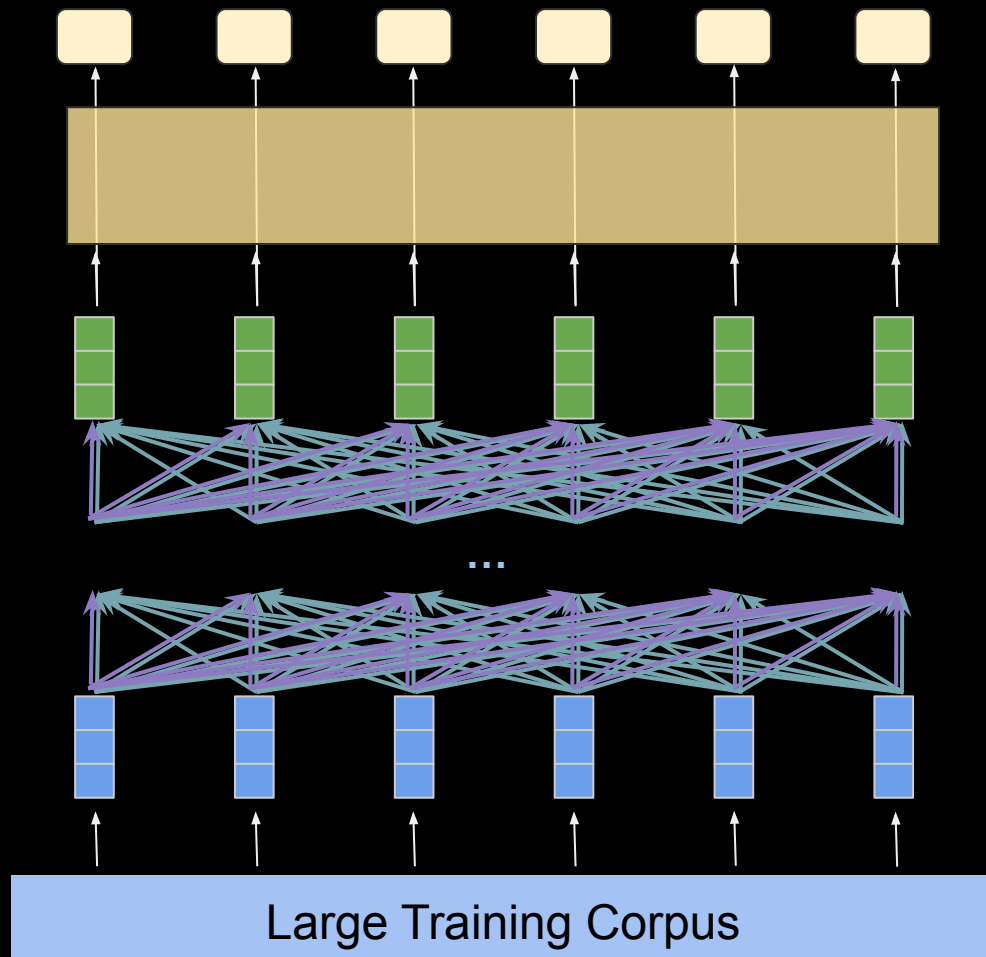
***optional layer(s) for task:***

***layer  $k-1$ :***  
*(taken as contextual embedding)*

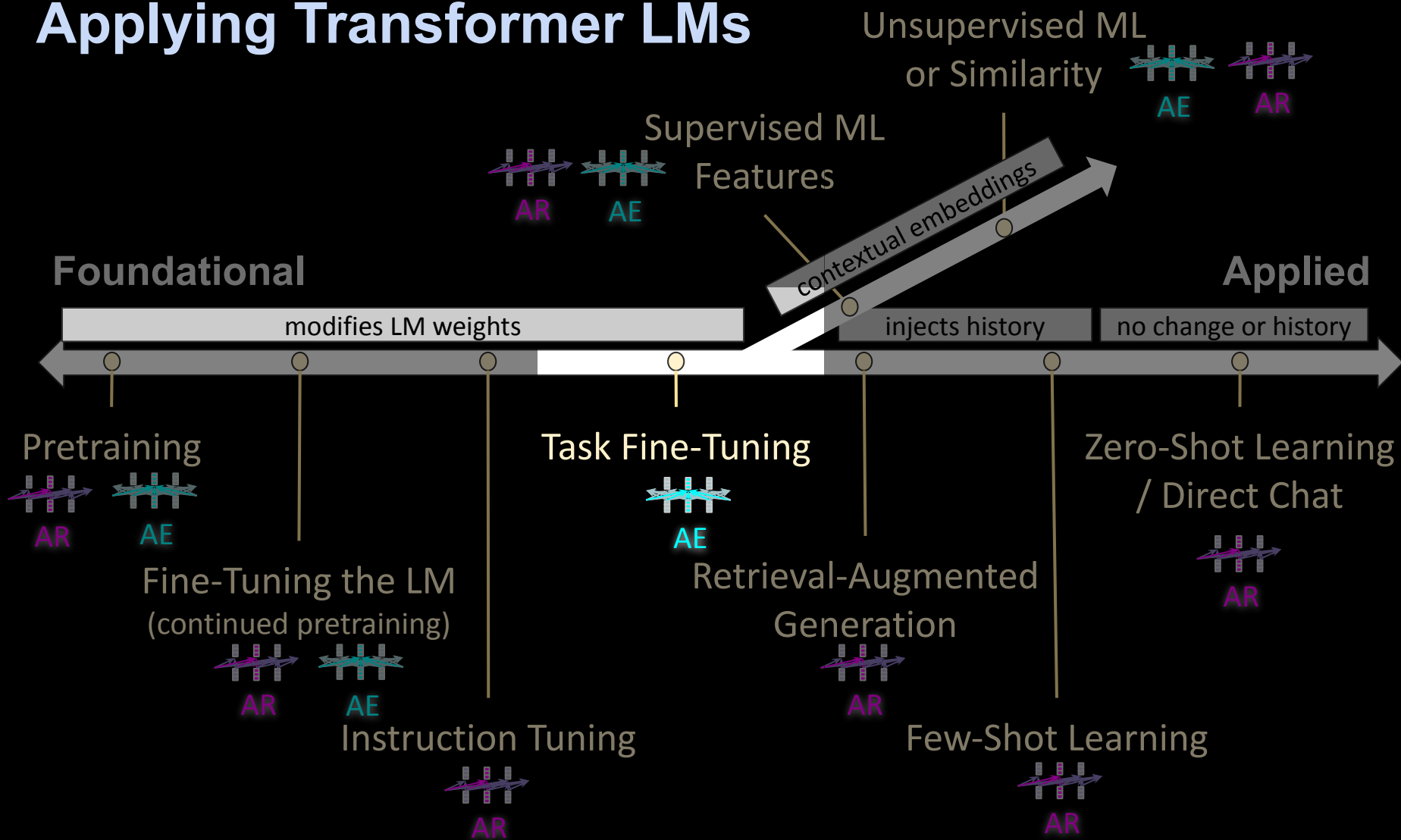
***layers 1 to  $k-2$ :***  
*(compose embeddings with context)*

***layer 0:***  
*(input: word-type embeddings)*

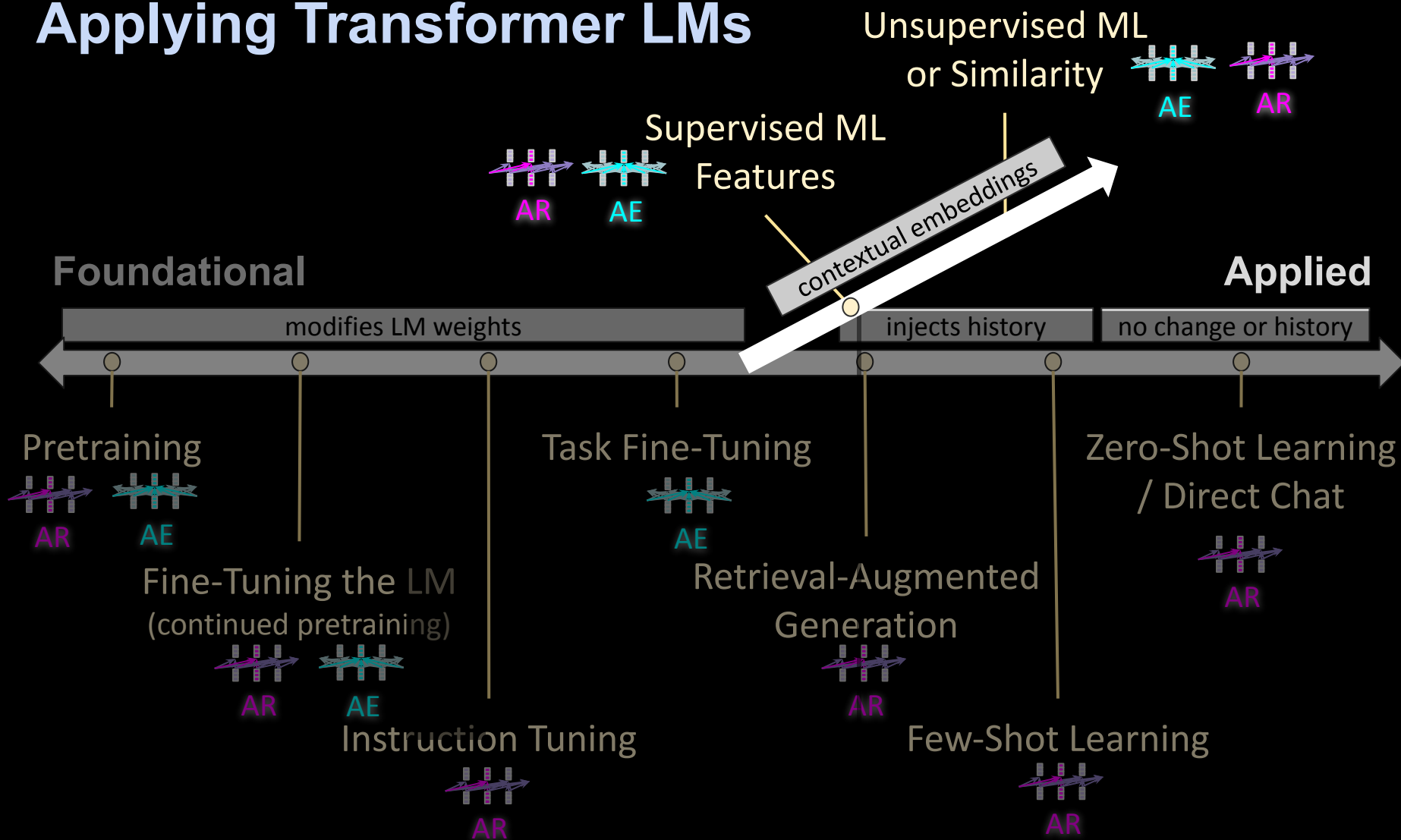
***sentence (sequence) input:***



# Applying Transformer LMs



# Applying Transformer LMs



# Contextual Embeddings: for Supervised ML; for Similarity (unsup)

*softmax for LM:*

*layer  $k$ :*

*(used for language modeling)*

*layer  $k-1$ :*

*(taken as contextual embedding)*

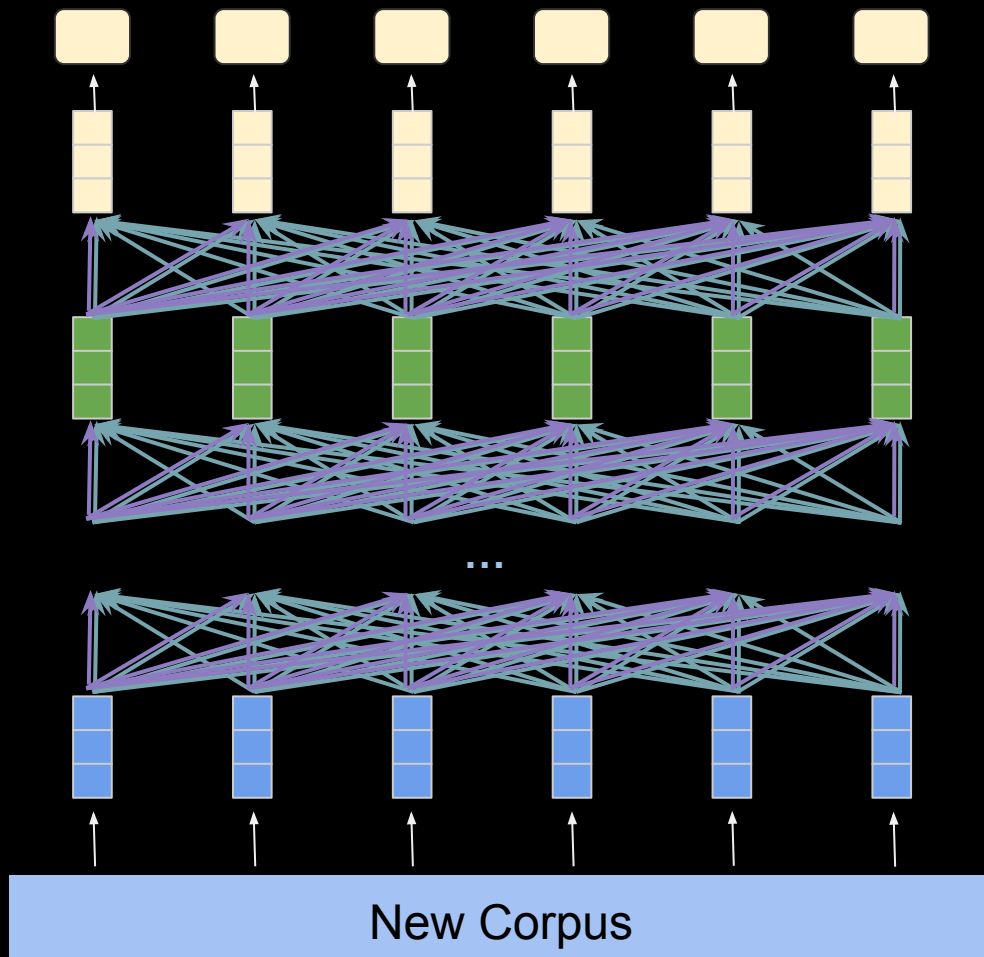
*layers 1 to  $k-2$ :*

*(compose embeddings with context)*

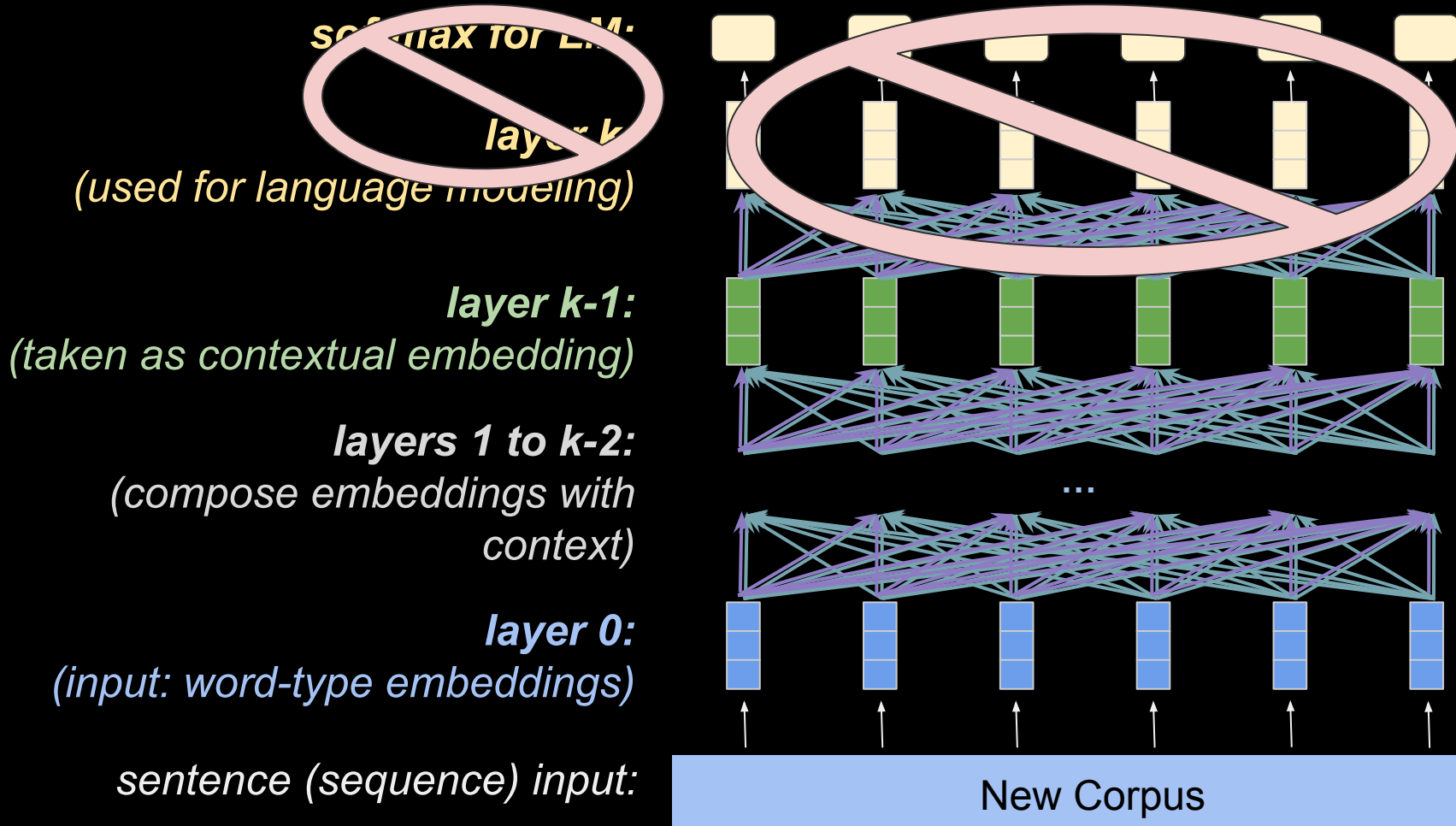
*layer 0:*

*(input: word-type embeddings)*

*sentence (sequence) input:*



# Contextual Embeddings: for Supervised ML; for Similarity (unsup)



# Contextual Embeddings: for Supervised ML

***classifier or regressor:***  
*(e.g. sentiment, topic classification, etc.)*

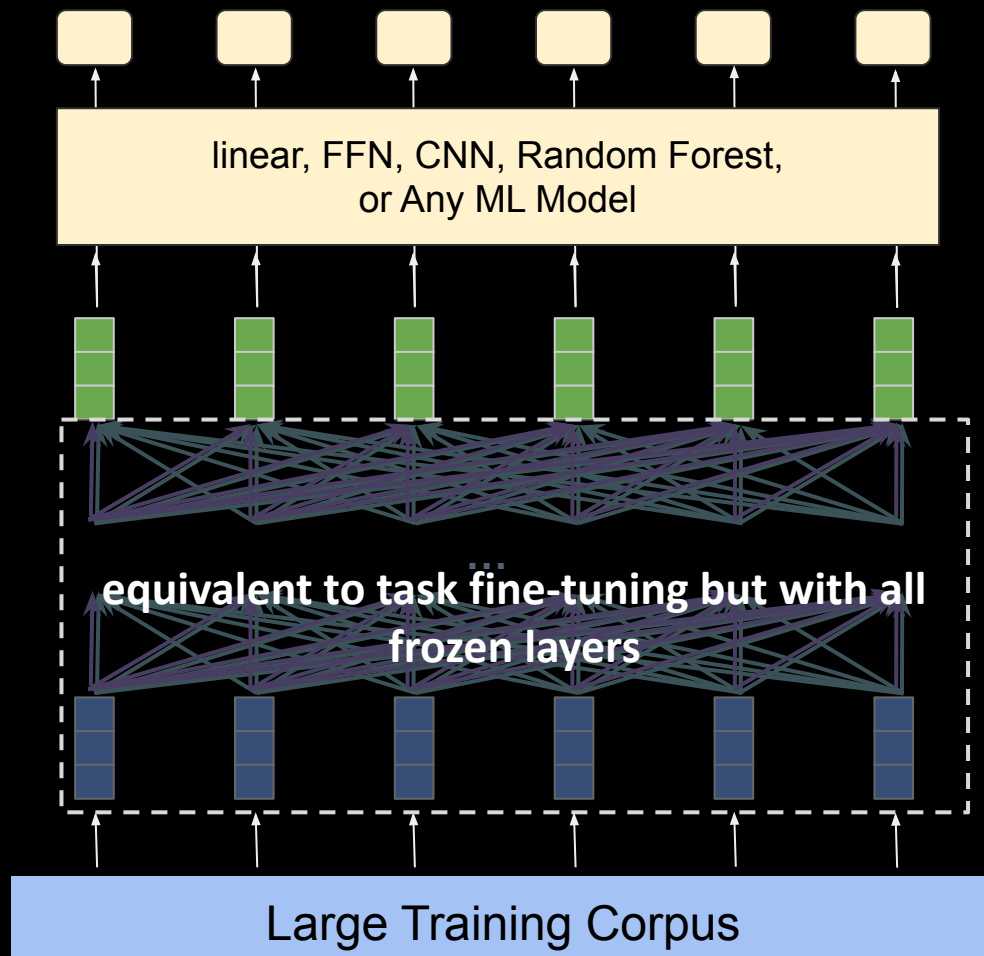
***layer(s) for task:***

***layer  $k-1$ :***  
*(taken as contextual embedding)*

***layers 1 to  $k-2$ :***  
*(compose embeddings with context)*

***layer 0:***  
*(input: word-type embeddings)*

***sentence (sequence) input:***



# Contextual Embeddings: for Similarity (unsup)

***classifier or regressor:***  
*(e.g. sentiment, topic classification, etc.)*

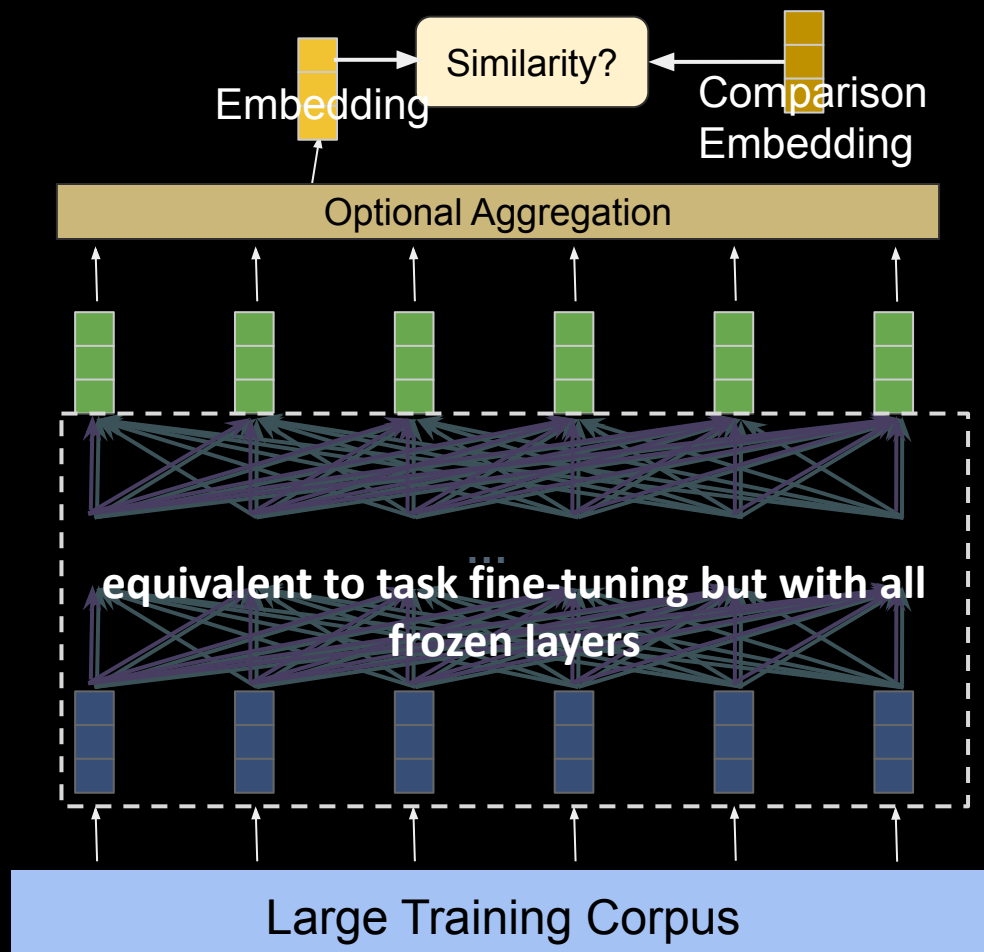
***layer(s) for task:***

***layer  $k-1$ :***  
*(taken as contextual embedding)*

***layers 1 to  $k-2$ :***  
*(compose embeddings with context)*

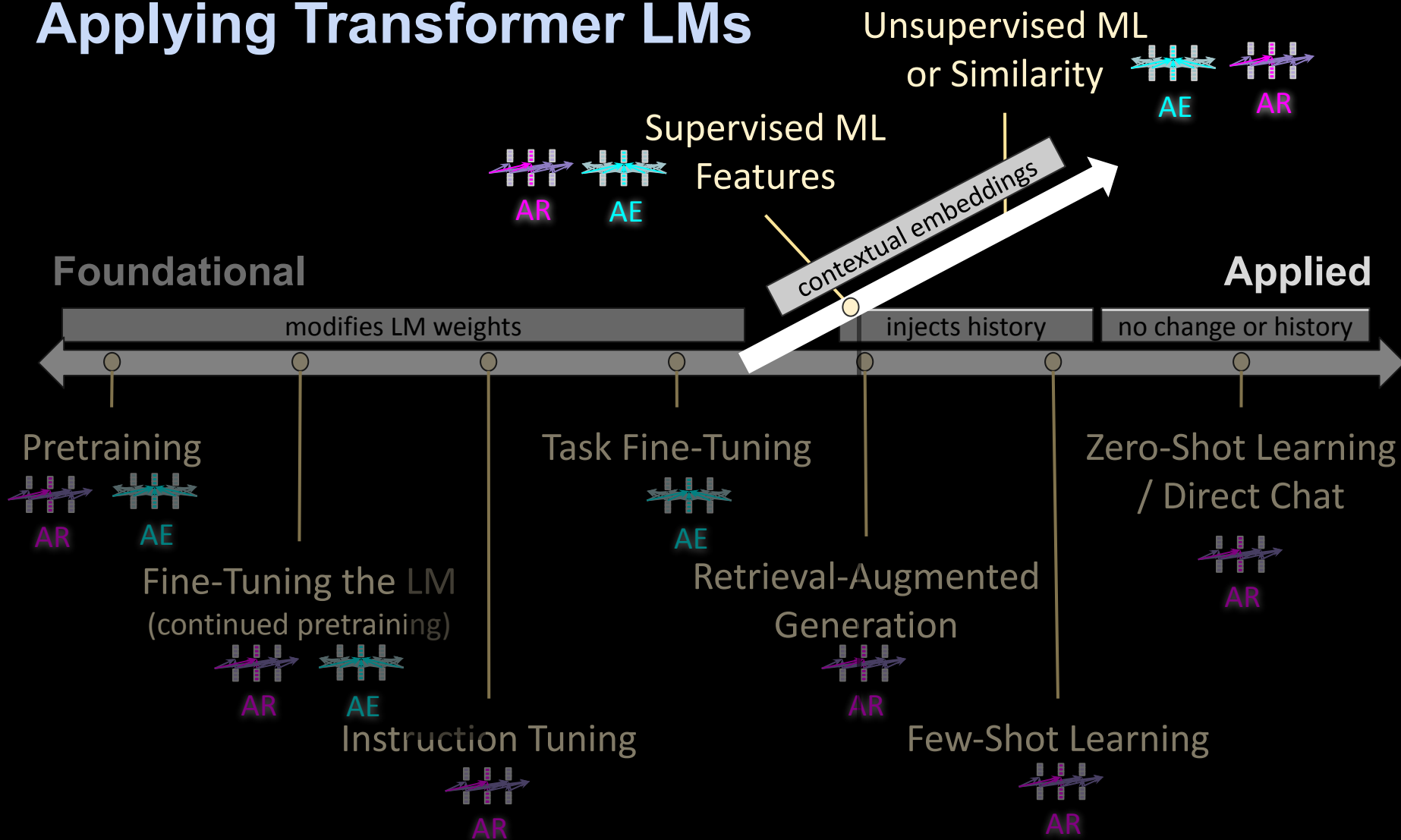
***layer 0:***  
*(input: word-type embeddings)*

*sentence (sequence) input:*

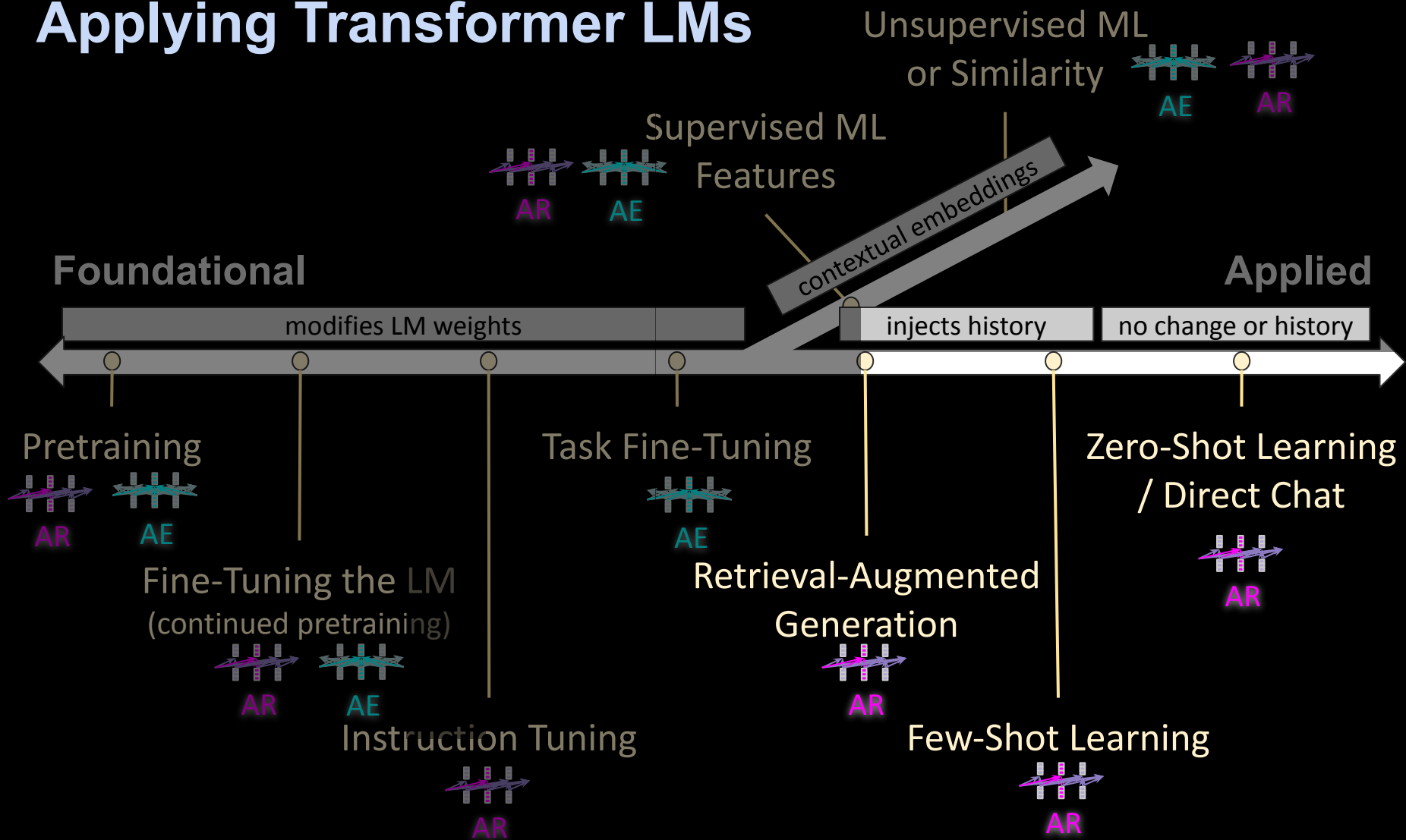




# Applying Transformer LMs



# Applying Transformer LMs



# RAG, Few-Shot, Zero-Shot

**softmax for LM:**

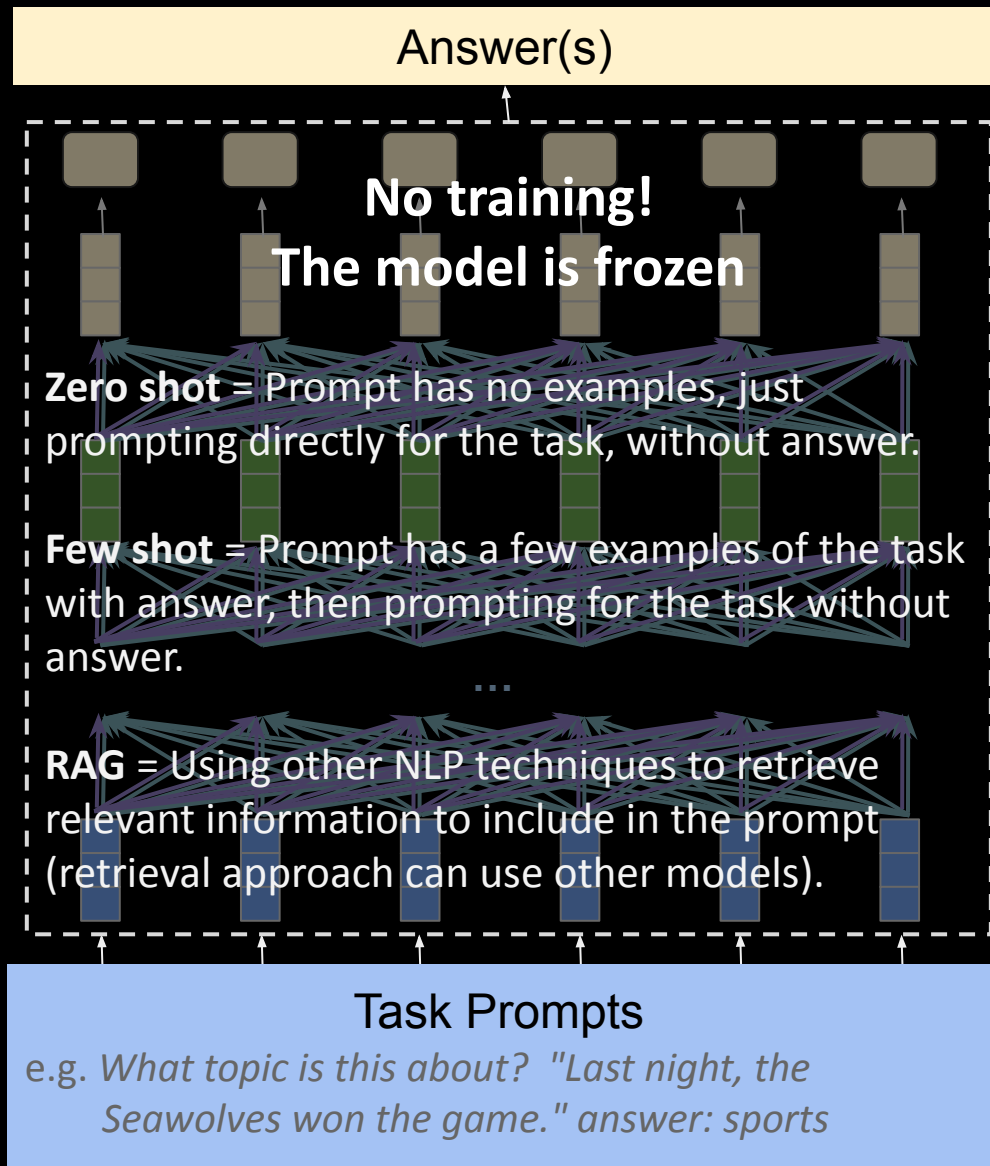
**layer  $k$ :**  
(used for language modeling)

**layer  $k-1$ :**  
(taken as contextual embedding)

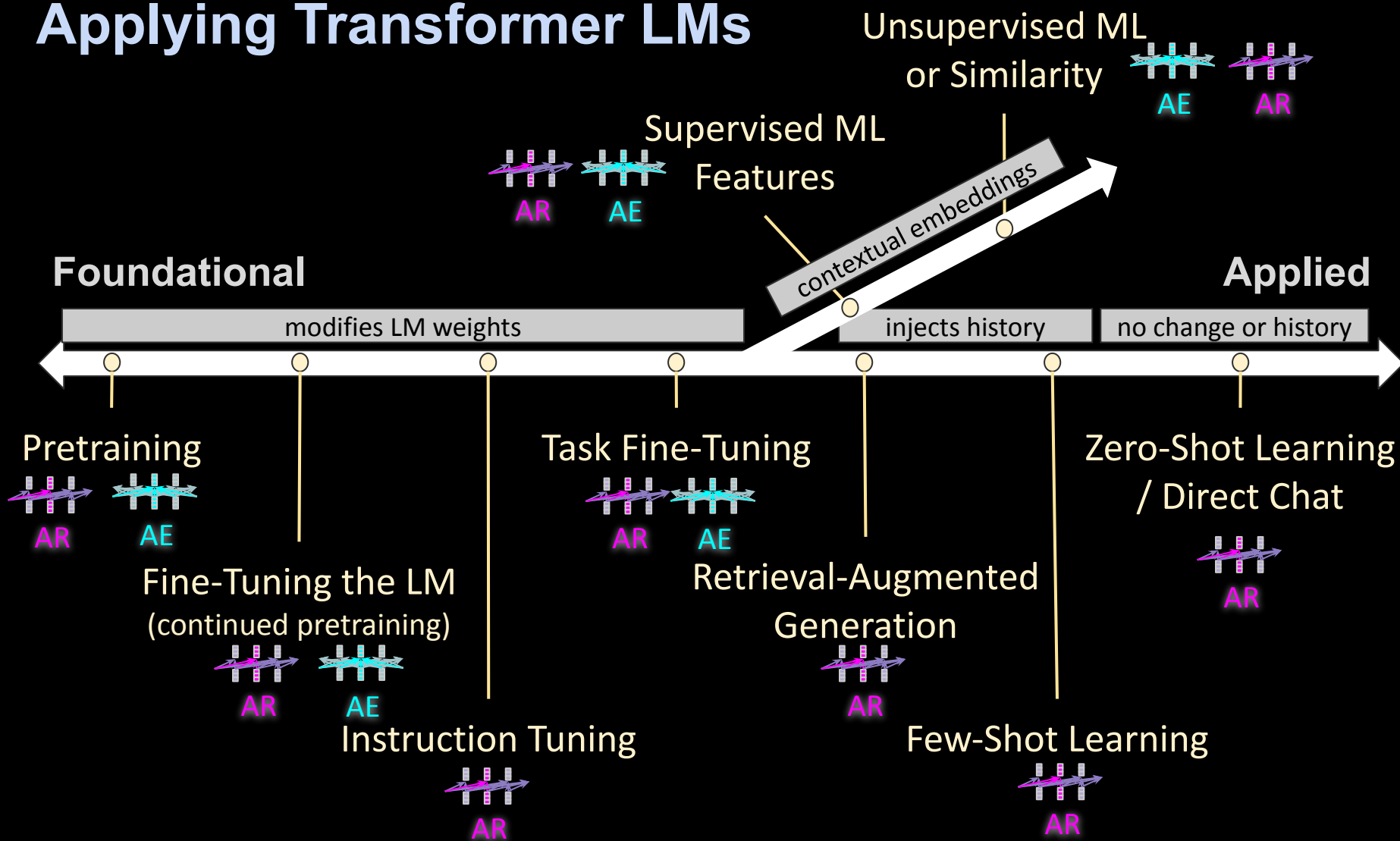
**layers 1 to  $k-2$ :**  
(compose embeddings with context)

**layer 0:**  
(input: word-type embeddings)

sentence (sequence) input:



# Applying Transformer LMs



# How to use an LM for Generation

- Greedy Search
- Beam Search
- Random Walk

# How to use an LM for Generation

- **Greedy Search**
- Beam Search
- Random Walk

Always take the most probable next word:

$$\hat{w}_t = \operatorname{argmax}_{w \in V} P(w | \mathbf{w}_{<t})$$

```
def generateGreedy(model, history='<s>'):
    vocabProbs = model.getNextProbs(history)
    history += argmax(vocabProbs)
                #word with maximum prob
    if history[-1] == '</s>': return history
    else: return generateGreedy(model, history)
```

# How to use an LM for Generation

- Greedy Search
- Beam Search
- Random Walk

Always take the most probable next word:

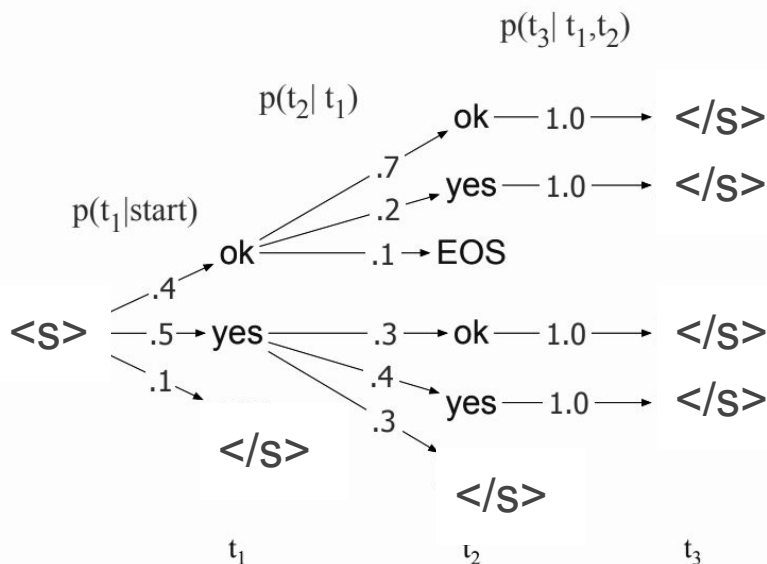
Problem:

$$p(' <s> ok ok </s> ')$$

$$=.28$$

$$p(' <s> yes yes </s> ')$$

$$=.20$$



**Figure 13.7** A search tree for generating the target string  $T = t_1, t_2, \dots$  from vocabulary  $V = \{\text{yes}, \text{ok}, \text{<s>}\}$ , showing the probability of generating each token from that state. Greedy search chooses *yes* followed by *yes*, instead of the globally most probable sequence *ok ok*.

# How to use an LM for Generation

- Greedy Search
- **Beam Search**
- Random Walk

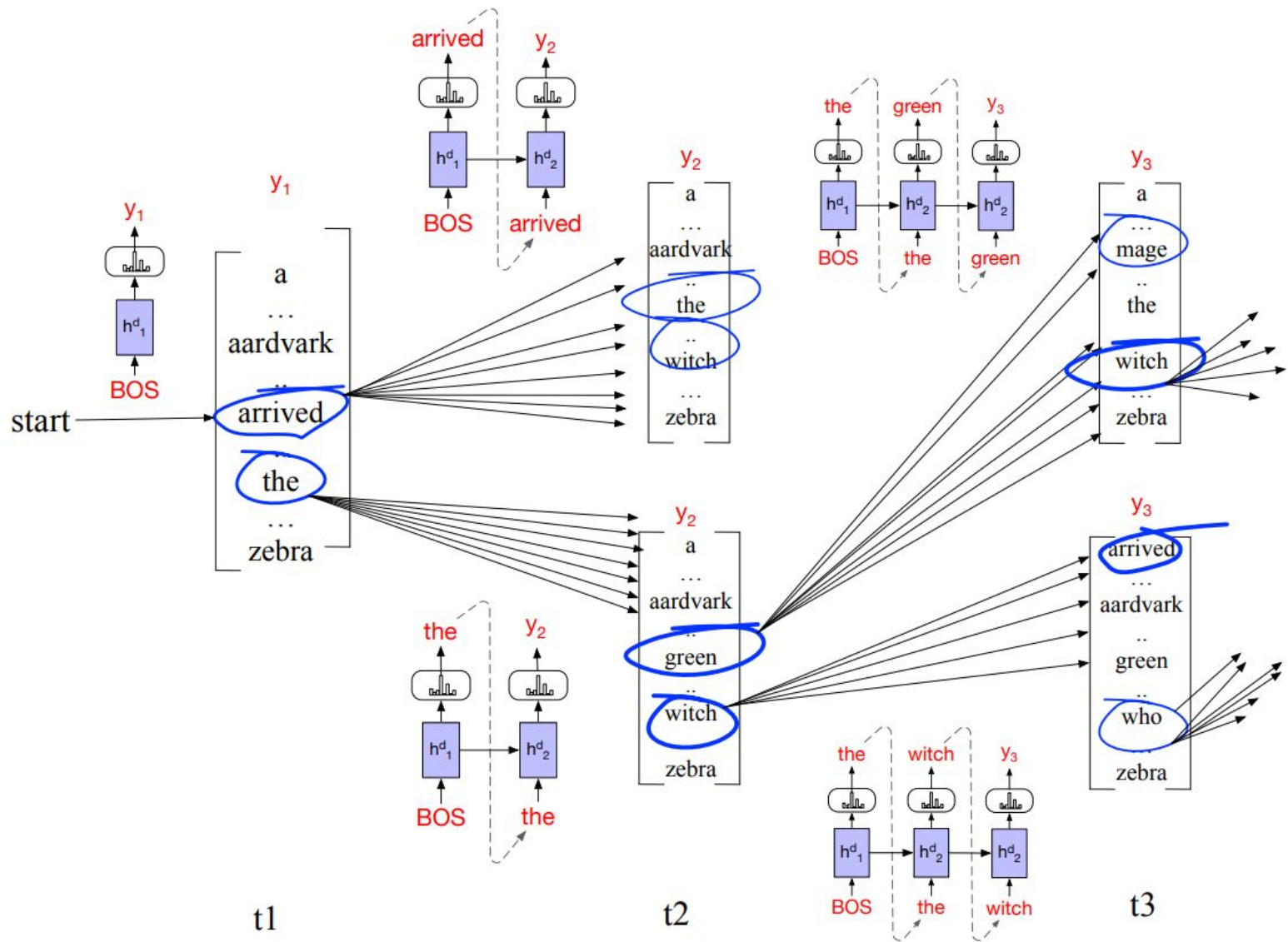
Evaluate among multiple sequences.

Restrict to consider the top  $k$  (*beam width*) most probable per step.

```
def generateBeam(model, history='<s>', init_prob=1, k=4):
    frontier = [(history, init_prob)]
    max_path = []
    max_path_p = -1.0
    while path, path_p in frontier:
        if path[-1] == "</s>": #current max
            if path_p > max_path_p:
                max_path = path
                max_path_p = path_p
        else:
            vocabProbs = model.getNextProbs(path)
            nextWPs = topK(vocabProbs, k)
            for w, p in nextWPs.items():
                frontier.append((path+w, path_p*p))
    return max_path, max_path_p
```







# How to use an LM for Gene

- Greedy Search
- **Beam Search**
- Random Walk

Evaluate among multiple sequences.

Restrict to consider the top  $k$  (*beam width*) most probable per step.

```
def generateBeam(model, history, k):
    frontier = [(history, 1.0)]
    max_path = []
    max_path_p = -1.0
    while path, path_p in frontier:
        if path[-1] == "</s>": #current potential end
            if path_p > max_path_p:
                max_path = path
                max_path_p = path_p
        else:
            vocabProbs = model.getNextProbs(path)
            nextWPs = topK(vocabProbs, k)
            for w, p in nextWPs.items():
                frontier.append((s+w, path_p*p))
    return max_path, max_path_p
```

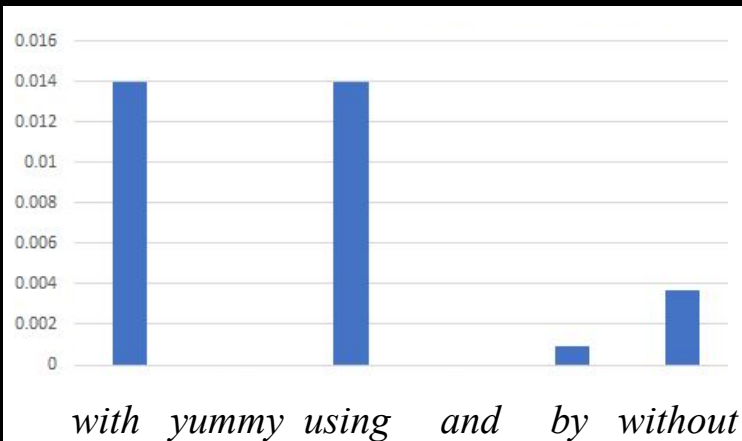
Disadvantage: Focuses on the most probable, which is the most typical. Results in very "average sounding" utterances.

# How to use an LM for Generation

- Greedy Search
- Beam Search
- **Random Walk**

```
def generateRandWalk(model, history='<s>'):  
    vocabProbs = model.getNextProbs(history)  
    history += multinomial.draw(vocabProbs)  
    #random multinomial draw by probs  
    if history[-1] == '</s>': return history  
    else: return generateRandWalk(model, history)
```

*Task: Estimate  $P(w_i | w_1, \dots, w_{i-1})$*   
:P(masked word given history)  
 *$P(\text{with} | \text{He ate the cake } \langle M \rangle) = ?$*



# How to use an LM for Generation

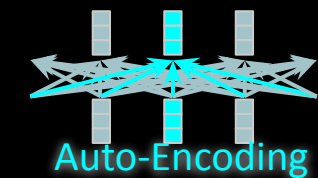
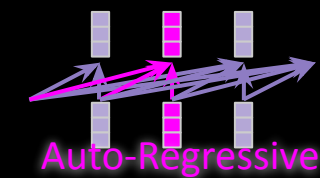
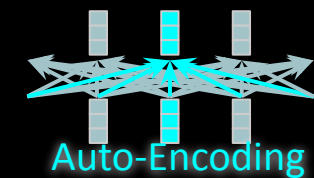
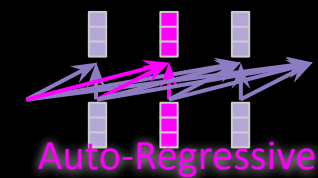
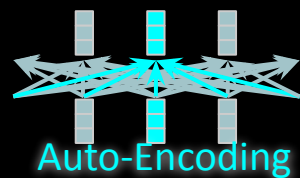
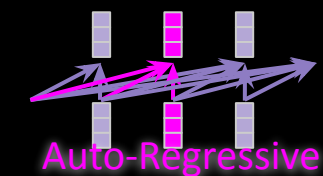
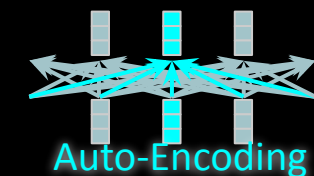
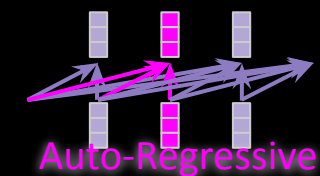
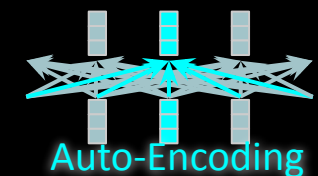
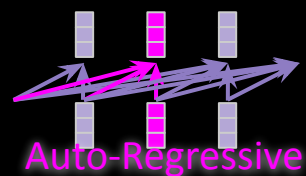
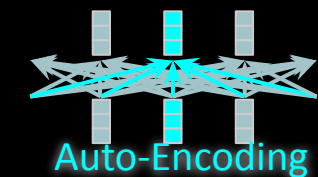
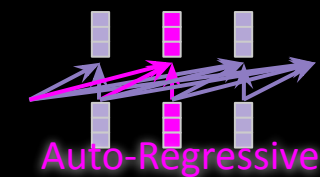
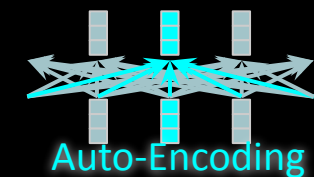
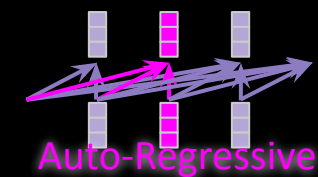
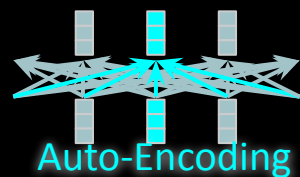
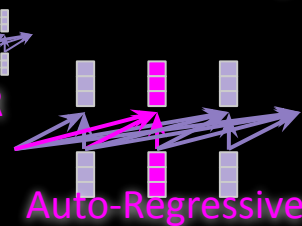
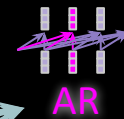
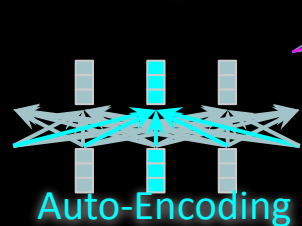
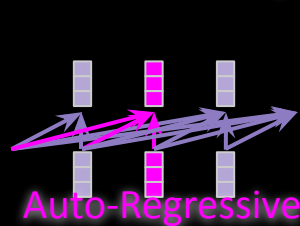
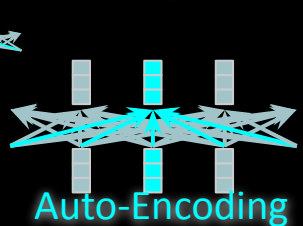
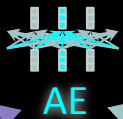
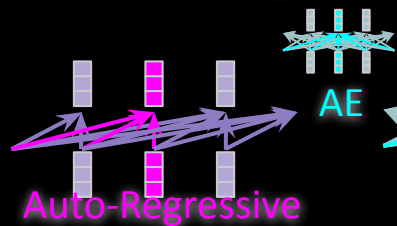
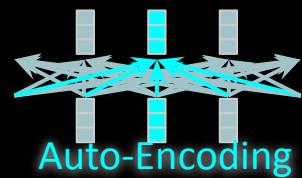
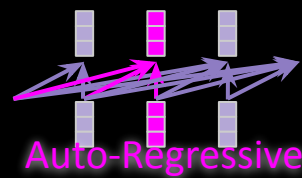
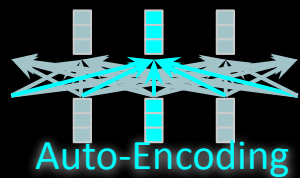
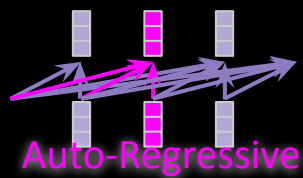
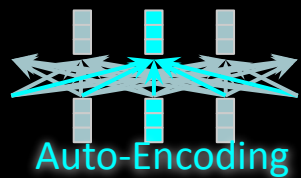
## Practical Points

- Use log probs for faster computation tracking maximums.
- Can normalize by length to not favor shorter sequences:

$$score(y) = \log P(y|x) = \frac{1}{t} \sum_{i=1}^t \log P(y_i | y_1, \dots, y_{i-1}, x) \quad (13.16)$$

- Combine beam and random walk for more novelty.

# **Supplemental Review Material**



# Linear Regression as DAG

How do Machine learning/ Deep learning frameworks represent these models?

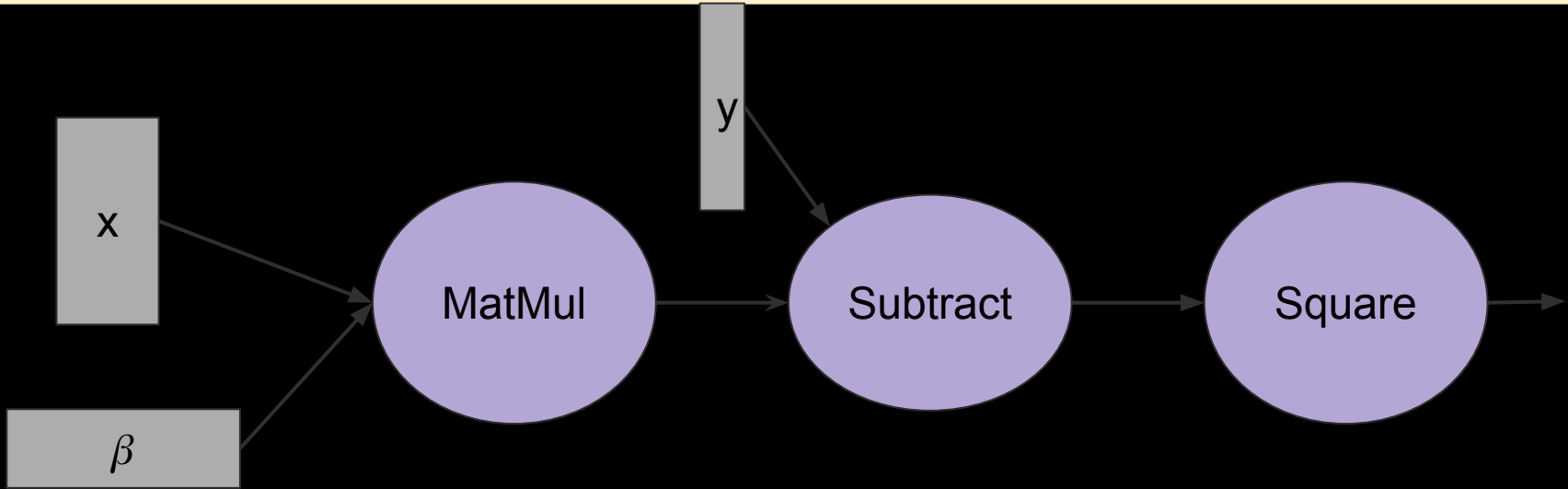


# Linear Regression as DAG

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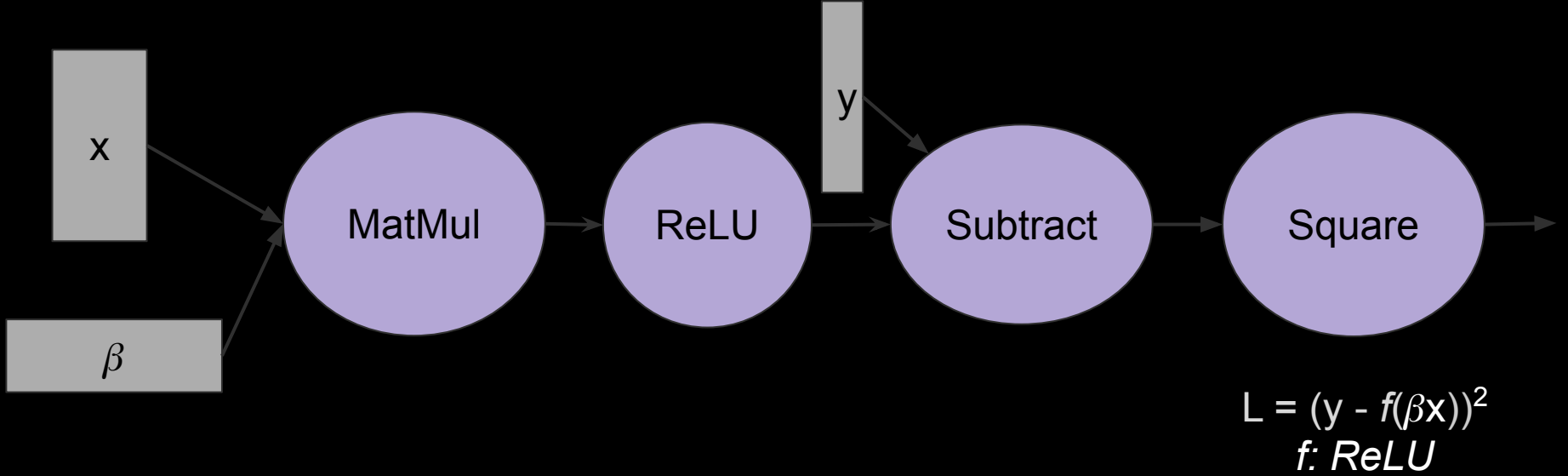
Computational Graph!

# Linear Regression as DAG

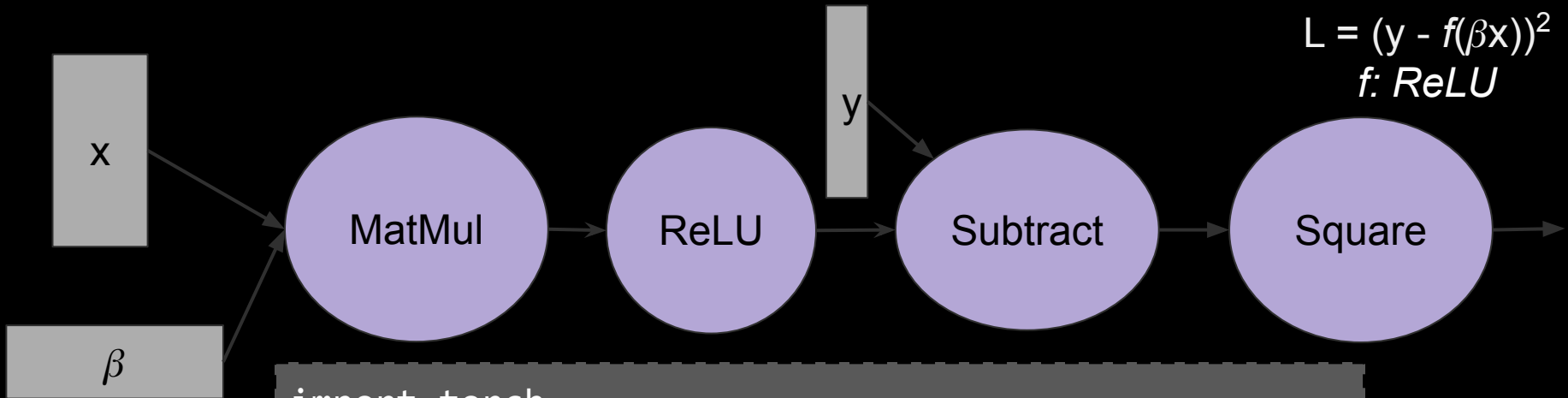


$$L = (y - \beta x)^2$$

# Linear Regression as DAG



# Linear Regression as DAG



```
import torch
from torch import nn

x = torch.Tensor(input)
beta = torch.random.randn(X.shape, 1)
z = torch.matmul(x, beta)
yhat = nn.functional.relu(z)
loss = nn.MSELoss(yhat, torch.Tensor(y))
```

# PyTorch Demo

Native Linear Regression Implementation ([Link](#))

Torch.nn Linear Regression Implementation ([Link](#))

# Linear Regression

Linear Regression:  $\hat{y} = \beta X$

Objective: *Learn  $w$ , such that  $(y - \beta X)^2$  is minimized*

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$$\beta_{opt} = (X^T X)^{-1} X^T y$$

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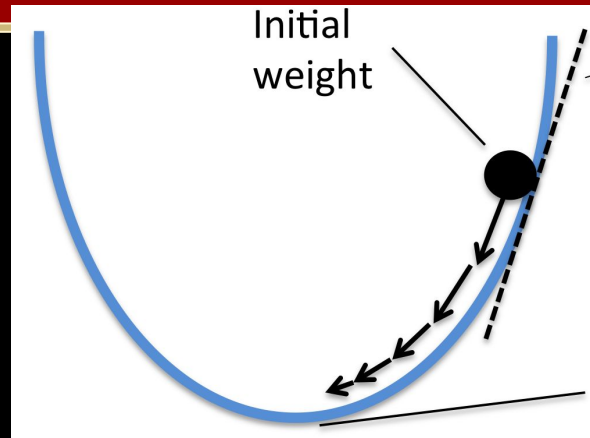
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# Numerical Gradient Approach

**Linear Regression:** Trying to find “betas” that minimize:


$$\beta^* = \operatorname{argmin}_{\beta} \{\sum_i (y_i - \hat{y}_i)^2\}$$

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$$\beta_{\text{new}} = \beta_{\text{old}} - \alpha * \text{grad}$$

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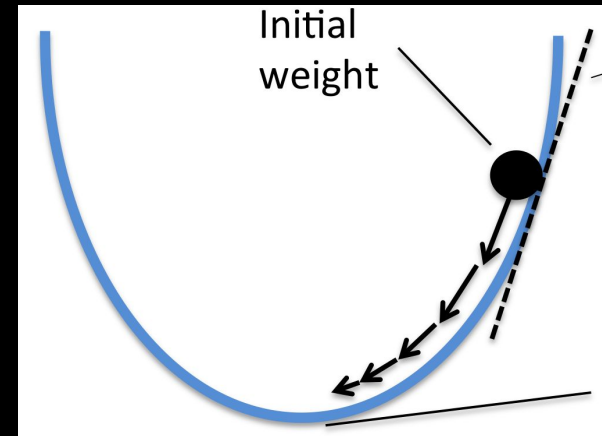
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$\alpha$ : Learning Rate





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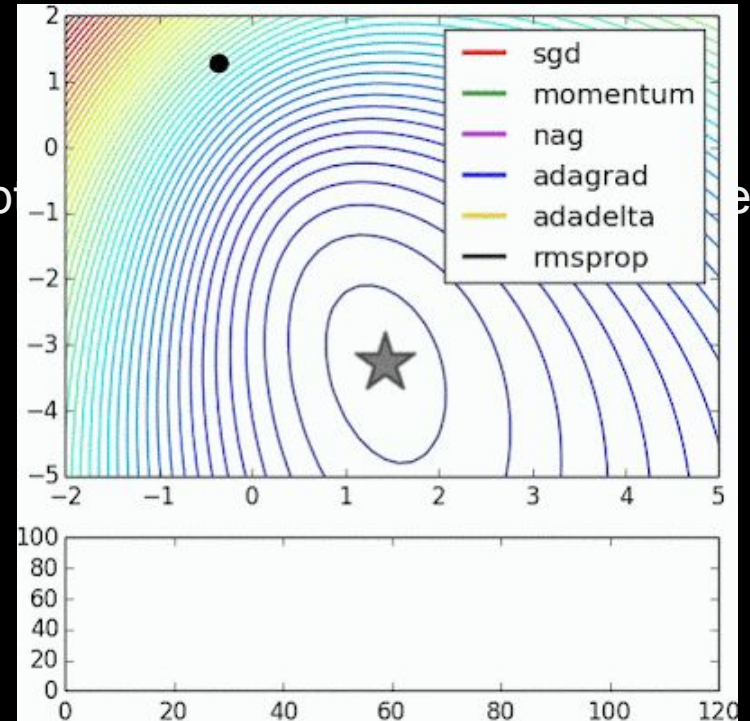
But there are other gradient descent based optimization methods which are better\*

# Numerical Gradient Approach

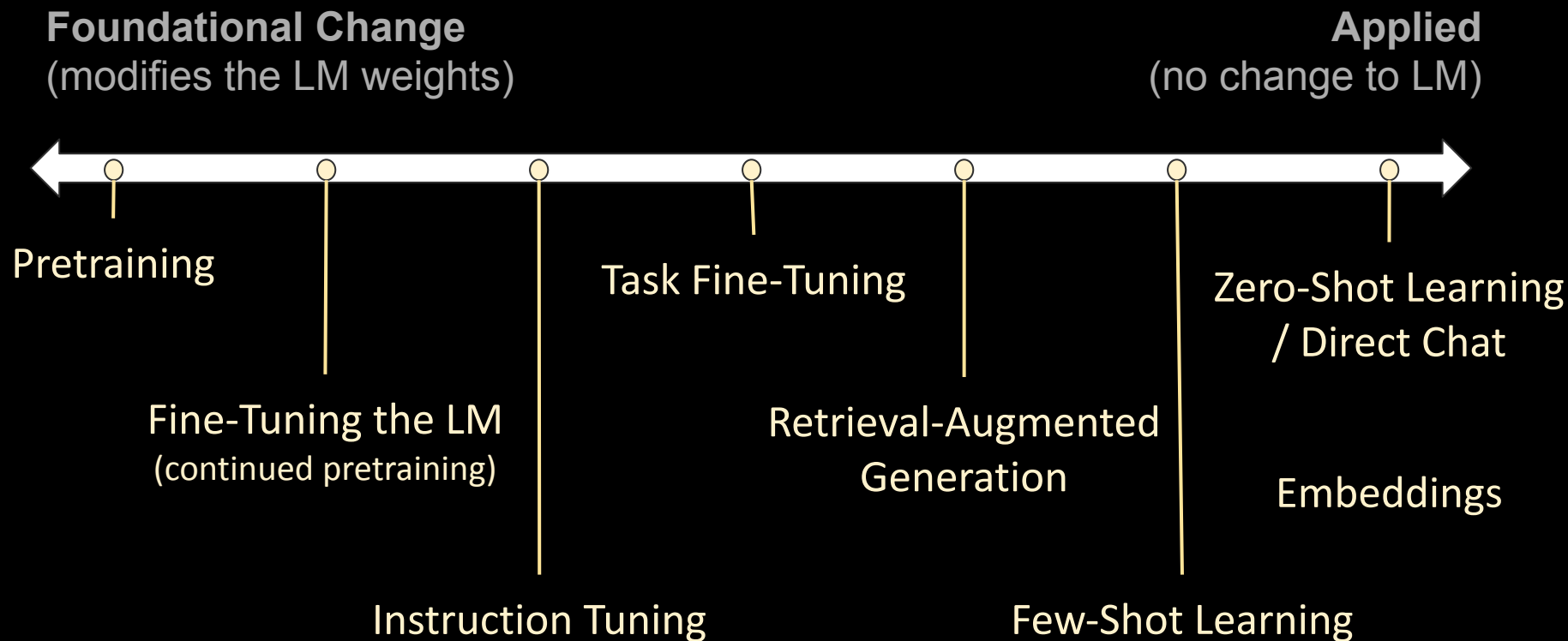
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# simpler version



# Pretraining; FTing the LM; Instruction Tuning

**softmax for LM:**

**layer  $k$ :**

*(used for language modeling)*

**layer  $k-1$ :**

*(taken as contextual embedding)*

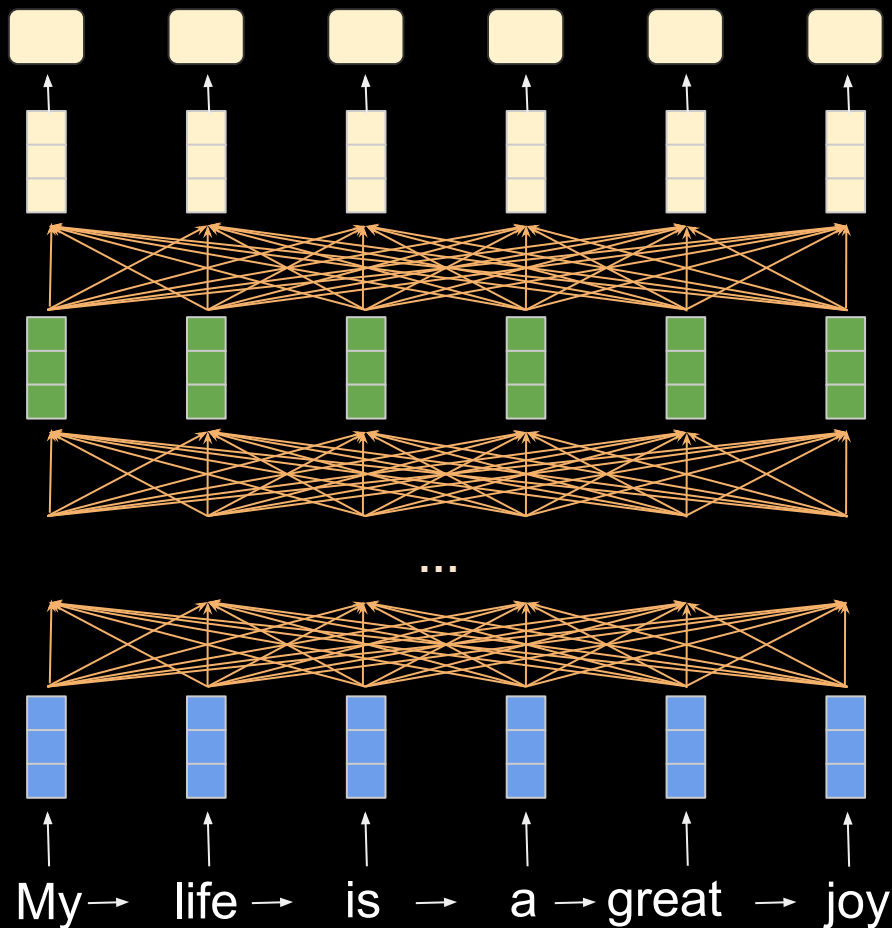
**layers 1 to  $k-2$ :**

*(compose embeddings with context)*

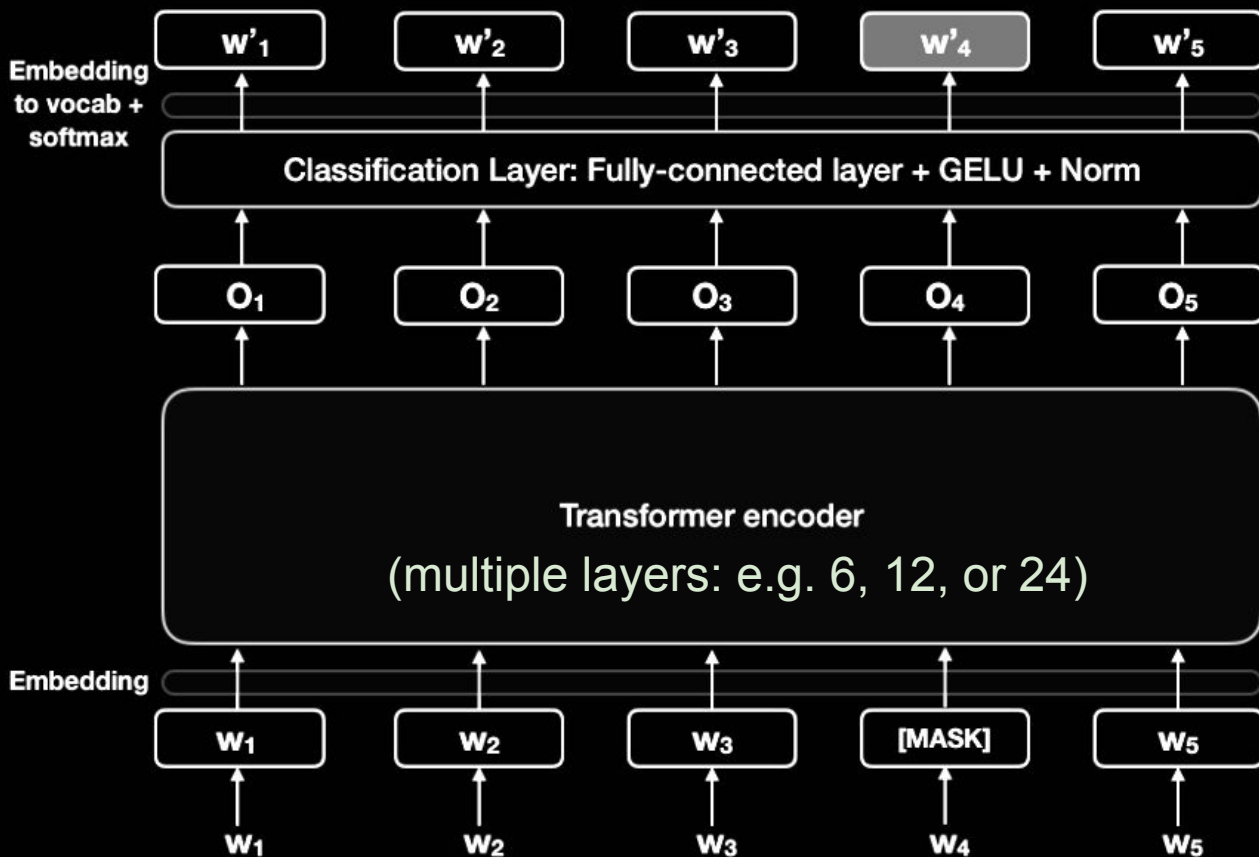
**layer 0:**

*(input: word-type embeddings)*

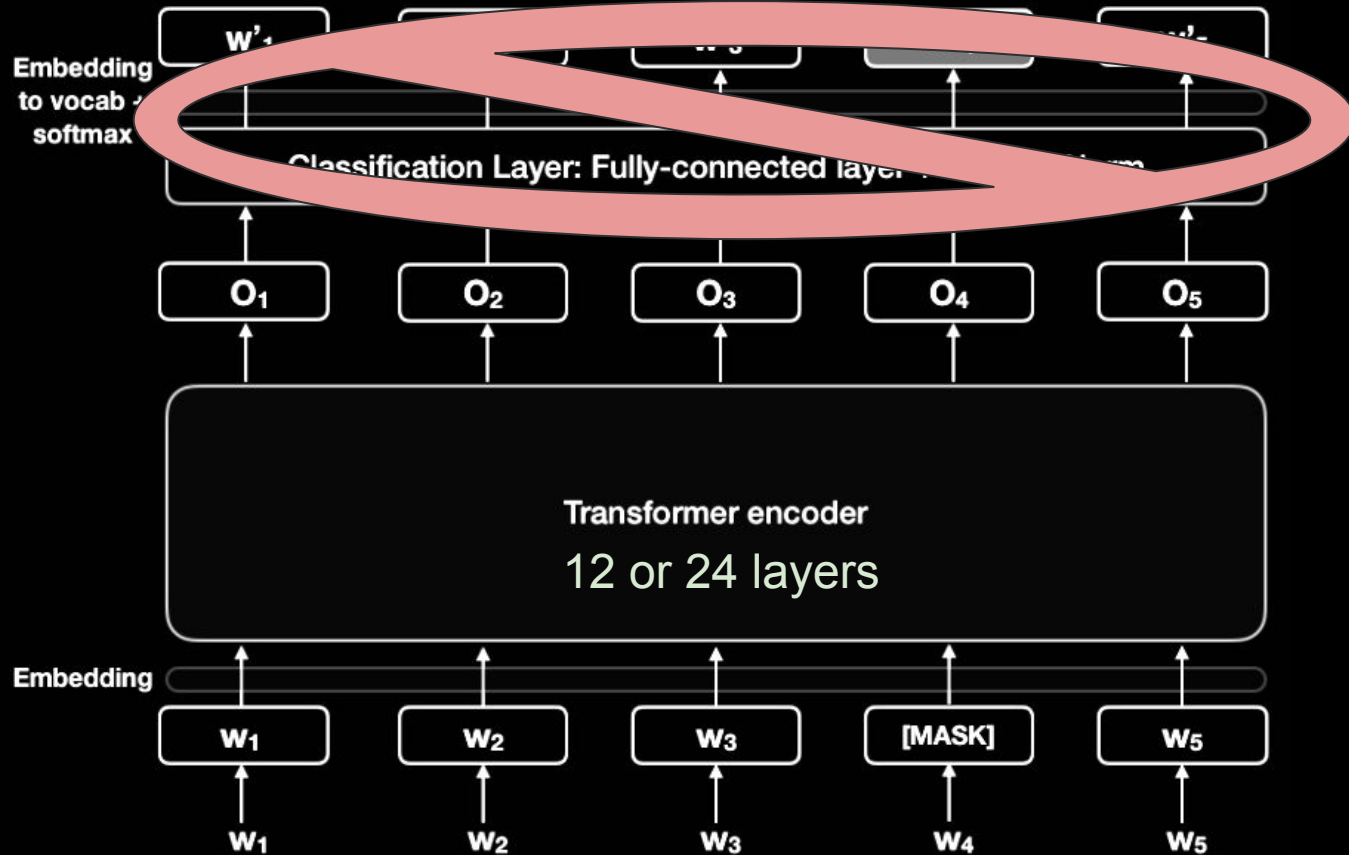
*sentence (sequence) input:*



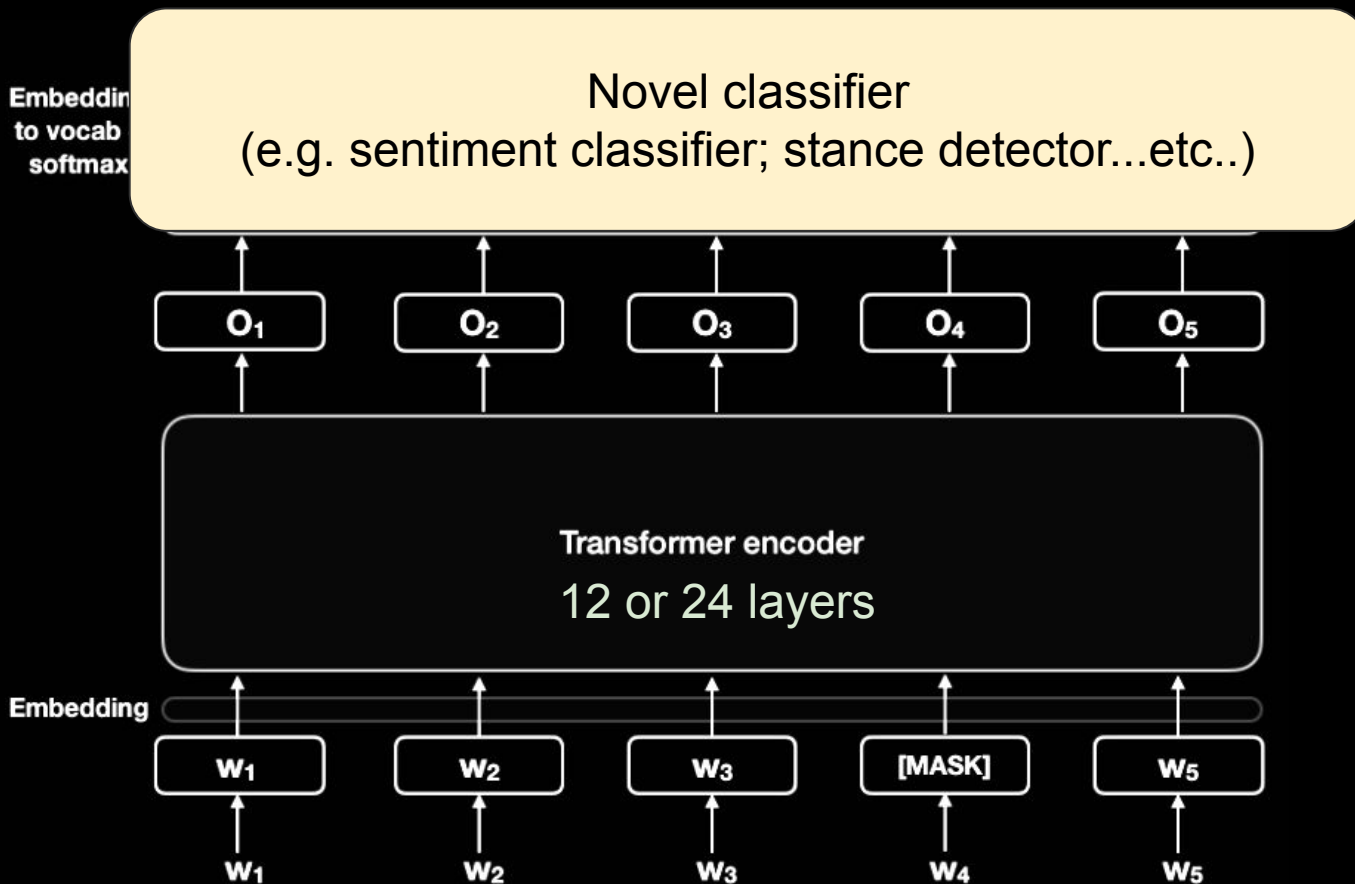
# Pre-training; Fine-tuning the LM; Instruction Tuning



# BERT: Pre-training; Fine-tuning



# BERT: Pre-training -> Task Fine-tuning





# BERT: Pre-training -> LM Fine-tuning

